Nuclear Equation of State for Compact Stars and Supernovae

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CompOSE and WG2+3 meeting

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This is not a conventional talk. I will not present the results of a particular work. The scope of this talk is to initiate & motivate the discussion on the topic of the nuclear EoS

The Four Topics of the WG2 as defined in the MOU of NewCompstar

- ❑ Nuclear EoS for Compact Stars & Supernovae
- ❑ Low-energy QCD & Super-dense matter
- ❑ Superfluidity & Superconductivity in Compact Stars
- ❑ Transport phenomena & Reaction rates for Compact Stars & Supernovae

Nuclear EoS for Compact Stars & Supernovae

One of the tasks of WG2 is:

Validate available EoS across different experiments & observations. The WG will define a protocol where the EoS, and the underlying interactions, are consistently checked and compared with nuclear experiments & observations of compact stars

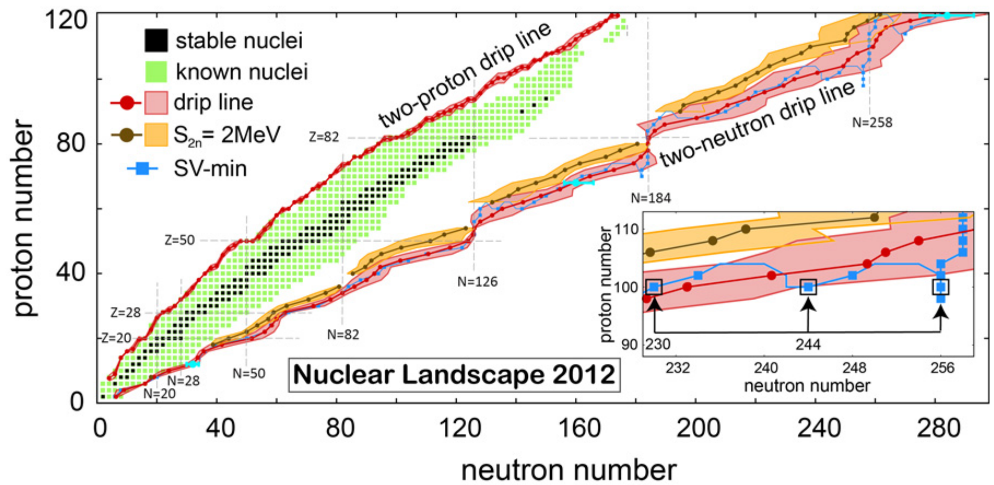
To such end we need to:

- ❖ Define where we are today (not an easy task)
  - search, compile & classify existing EoS
- ❖ Define where we want to go tomorrow (even more difficult)
  - Perhaps perform a collective effort to construct reliable EoSs to be used in astrophysical applications ? Partly done with CompOSE

What do we know to build the Nuclear EoS ?

J. Erler et al., Nature 486, 509 (2012)

- Masses, radii & other properties of ~ 3200 isotopes



- Around ρ_0 the nuclear EoS can be characterized by few isoscalar & isovector parameters

$$\frac{E}{A}(\rho,\beta) = E_0 + \frac{1}{2}K_0 x^2 + \frac{1}{6}Q_0 x^3 + \left(E_{sym} + Lx + \frac{1}{2}K_{sym}x^2 + \frac{1}{6}Q_{sym}x^3\right)\beta^2 + \cdots$$

$$x = \frac{\rho - \rho_0}{3\rho_0} \quad , \quad \beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

❖ Isoscalar parameters

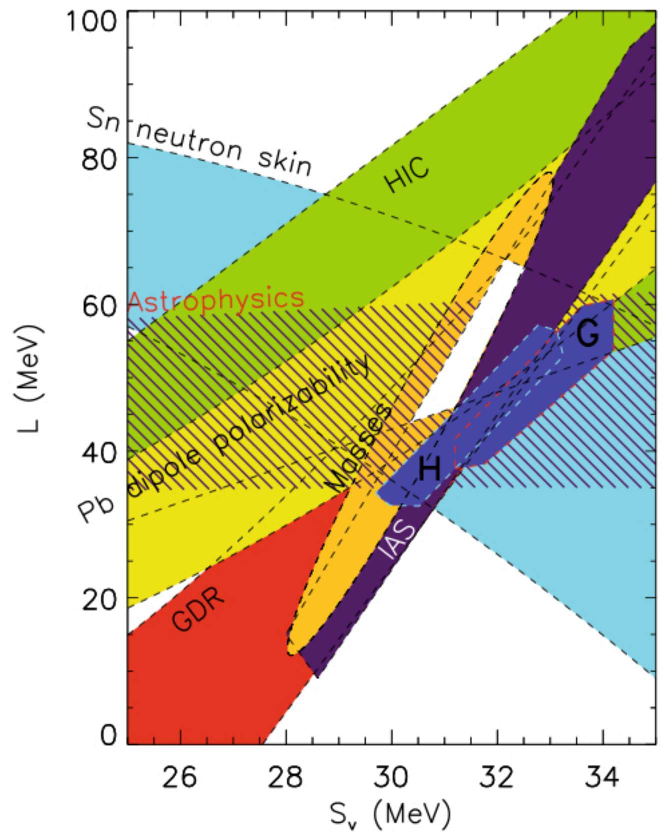
$$E_0 \approx -16 \quad MeV \quad , \quad K_0 = 9\rho_0^2 \frac{\partial^2 E_{IS}(\rho)}{\partial \rho^2}\bigg|_{\rho=\rho_0} \approx 240 \pm 20 \quad MeV \quad , \quad Q_0 = 27\rho_0^3 \frac{\partial^3 E_{IS}(\rho)}{\partial \rho^3}\bigg|_{\rho=\rho_0} \approx -500 \div 300 \quad MeV$$

❖ Isovector parameters

Less certain. Large variation of the prediction of the different models

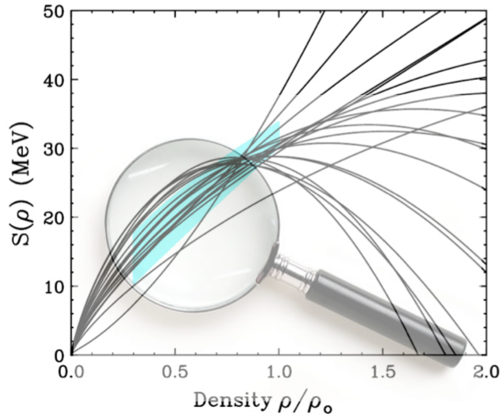
$$E_{sym} = \frac{1}{2}\frac{\partial^2 E/A}{\partial \beta^2}\bigg|_{\beta=0} \quad , \quad L = 3\rho_0 \frac{\partial E_{IV}}{\partial \rho}\bigg|_{\rho=\rho_0}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{IV}}{\partial \rho^2}\bigg|_{\rho=\rho_0} \quad , \quad Q_{sym} = 27\rho_0^3 \frac{\partial^3 E_{IV}}{\partial \rho^3}\bigg|_{\rho=\rho_0}$$



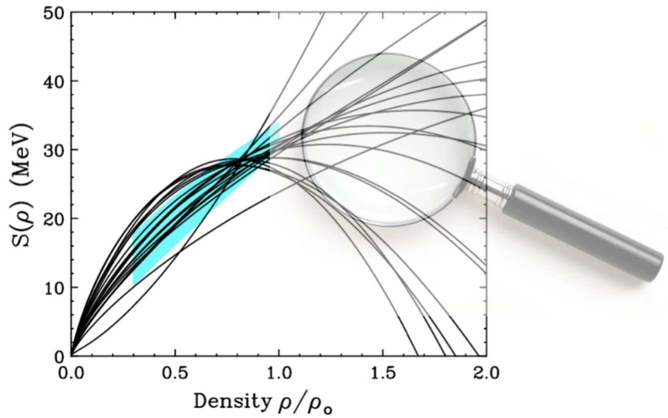
J. M. Lattimer & A. W. Steiner, EPJA 50, 40 (2014)

Symmetry Energy Sensitive Observables



- Sub-saturation densities

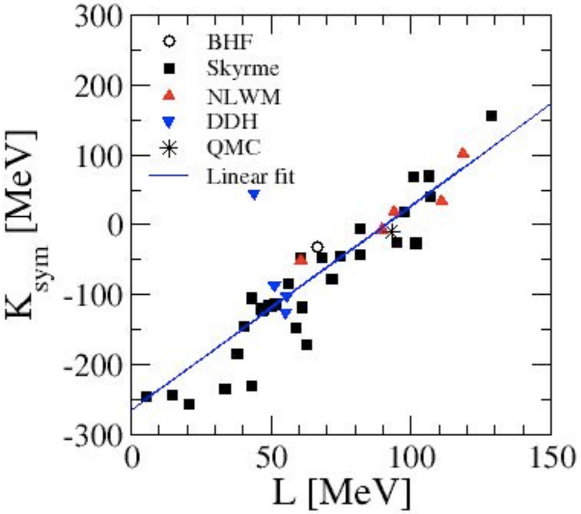
✓ Neutron skin thickness in heavy nuclei
✓ Giant & pygmy resonances in neutron-rich nuclei
✓ n/p & $t/^3He$ ratios in nuclear reactions
✓ Isospin fragmentation & isospin scaling in nuclear multi-fragmentation
✓ Neutron-proton correlation functions at low relative momenta
✓ Isospin diffusion/transport in heavy ion collisions
✓ Neutron-proton differential flow

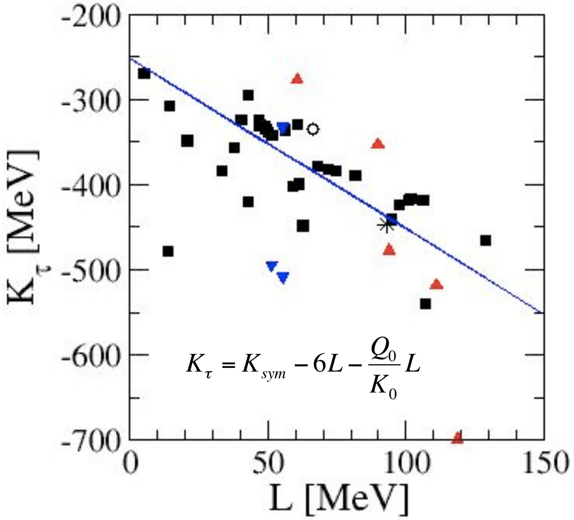


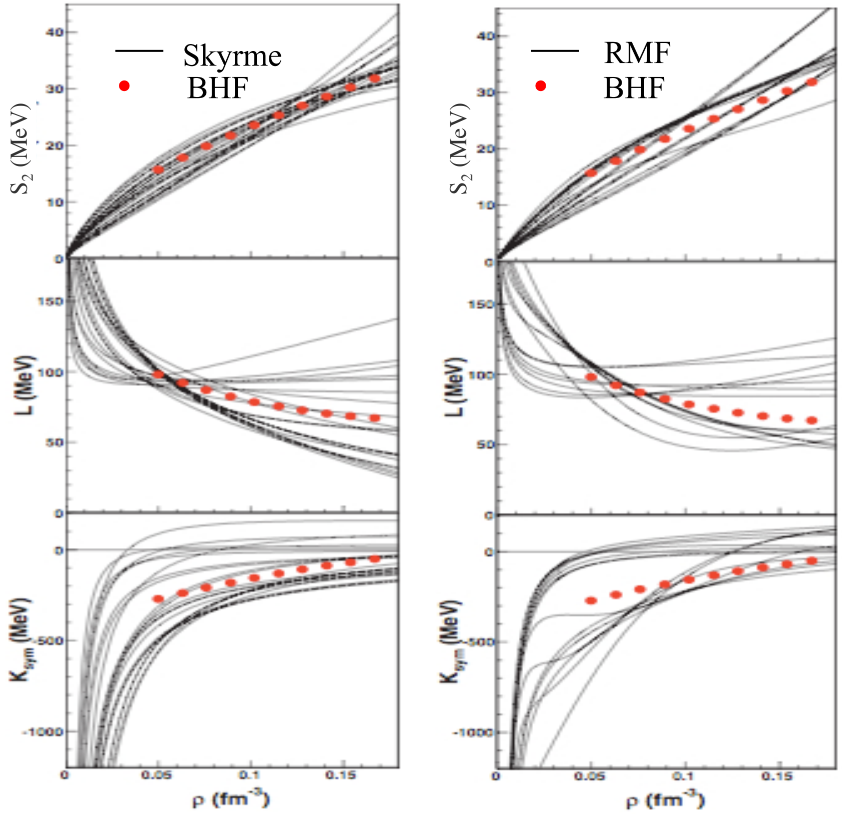
- Supra-saturation densities

✓ π^-/π^+ & K^-/K^+ ratios in heavy ion collisions
✓ Neutron-proton differential transverse flow
✓ n/p ratio of squeezed out nucleons perpendicular to the reaction plane
✓ Nucleon elliptic flow at high transverse momenta

Correlation of K_{sym} & K_τ with L





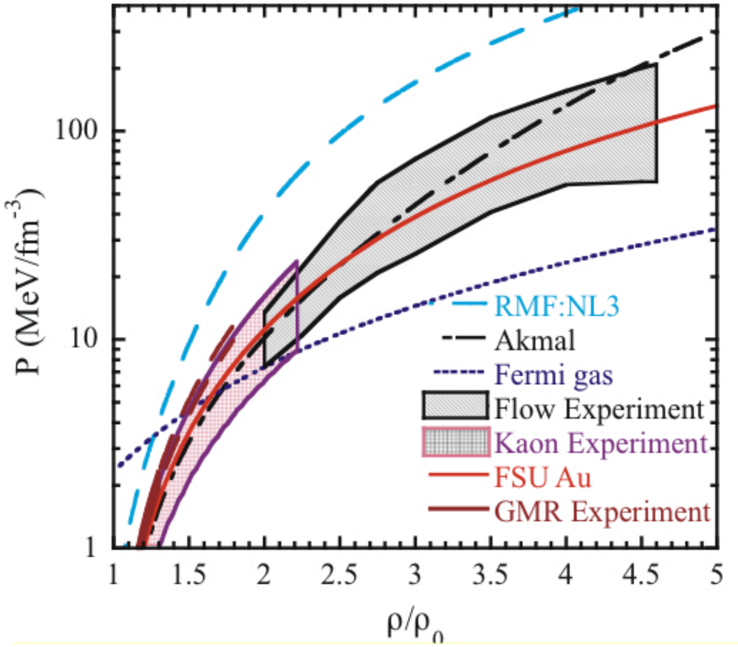


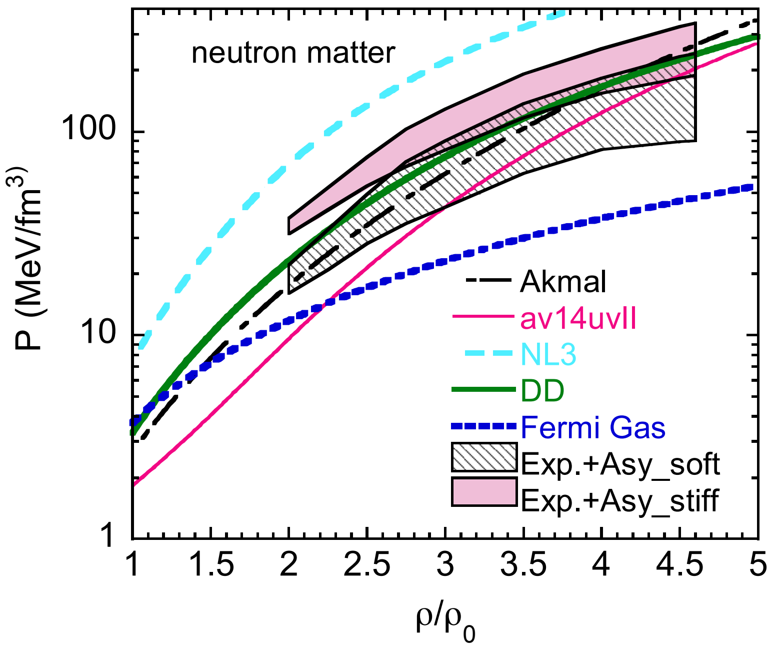
✓ E_{sym}: crossing at $\rho \sim 0.11$ fm^{-3}, $E_{sym}(0.11)$~24±4 MeV
(expected from finite nuclei constraints at $\rho<\rho_0$)

✓ L : tendency to cross at $\rho \sim \rho_0/3$

✓ K_{sym}: no crossing observed

Constraints of the Nuclear EoS from HIC



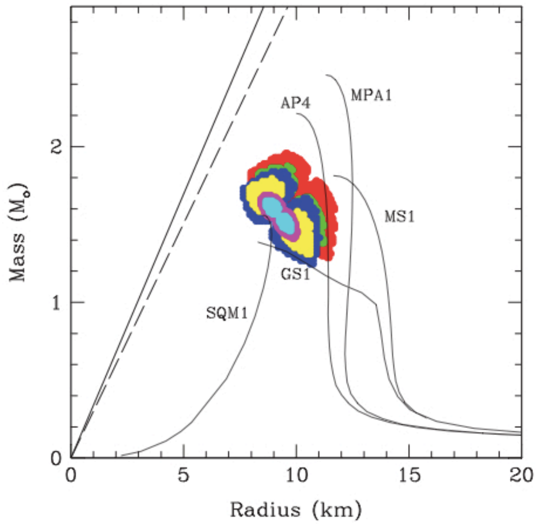


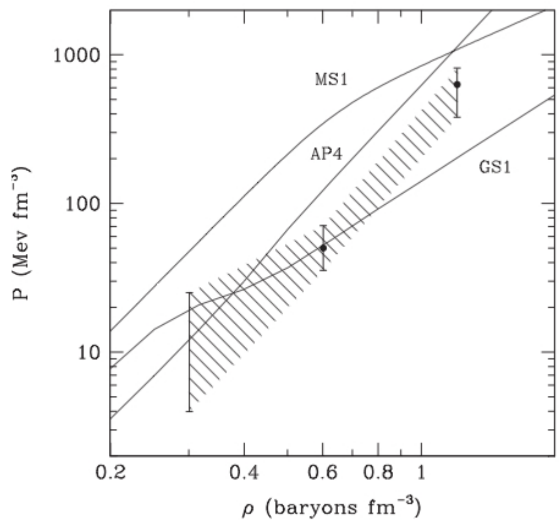
- ❖ Collective flow constraints confirms the softening of the EoS at high densities
- ❖ Constraints from kaon production are consistent with the flow constraints and bridge gap to GMR constraints
- ❖ Symmetry energy dominates the uncertainty in the neutron matter EoS

P. Danielewicz et al., Science 298, 1592 (2002)

Astrophysical determination of the Nuclear EoS

✧ Piecewise polytropic EoS above ρ_0 from mass-radius relation of 3 type-I X-ray bursts





- SLy below ρ_0
- Piecewise poytropic above ρ_0

$$\rho_{i-1} < \rho \le \rho_i, \quad \varepsilon = \alpha_i \rho + \beta_i \rho^{\Gamma_i}, \quad P = (\Gamma_i - 1)\beta_i \rho^{\Gamma_i}$$

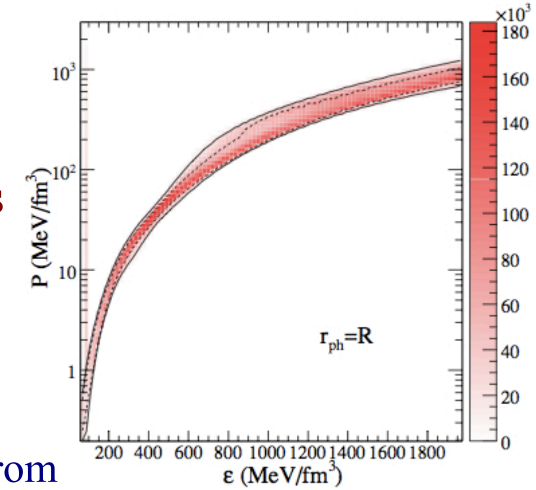
$\log P_0$ $(0.37\rho_{ns})$	$\log P_1$ $(1.85\rho_{ns})$	$\log P_2$ $(3.7\rho_{ns})$	$\log P_3$ $(7.4\rho_{ns})$
-0.64	[0.6–1.4]	$1.70^{+0.15}_{-0.15}$	$2.8^{+0.1}_{-0.2}$

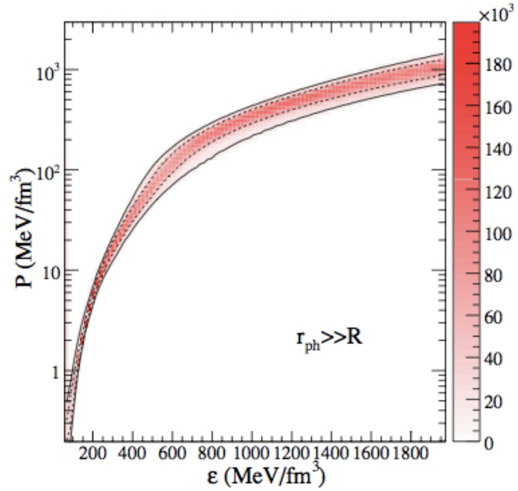
F. Ozel & D. Psaltis, PRD 80, 103003 (2009)
F. Ozel, G. Baym & T. Guver, PRD 82, 101301(R) (2010)

Astrophysical determination of the Nuclear EoS

✧ Nuclear parameters determined in a Bayesian data analysis of:

- ❖ 3 type-I X-ray burst
- ❖ 3 transient low mass X-ray binaries
- ❖ Cooling of 1 isolated NS, RX J1856-3754





Parameters in the range expected from nuclear systematics & lab. experiments

Quantity	Lower Limit	Upper Limit
K (MeV)	180	280
K' (MeV)	−1000	−200
S_v (MeV)	28	38
γ	0.2	1.2
n_1 (fm^{-3})	0.2	1.5
n_2 (fm^{-3})	0.2	2.0
ε_1 (MeV fm^{-3})	150	600
ε_2 (MeV fm^{-3})	ε_1	1600

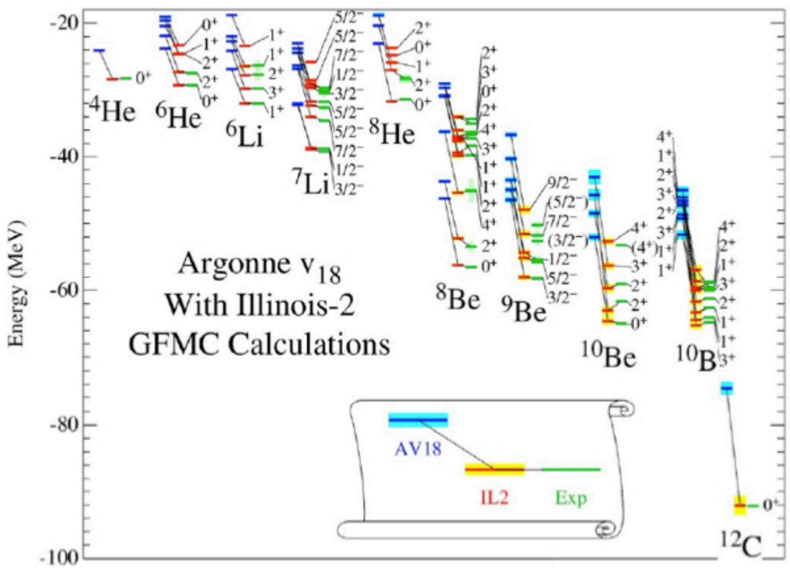
$$\varepsilon = n_B \left\{ m_B + B + \frac{K}{18}(u-1)^2 + \frac{K'}{162}(u-1)^3 + (1-2x)^2[S_k u^{2/3} + S_p u^{\gamma}] + \frac{3}{4}\hbar c x (3\pi^2 n_b x)^{1/3} \right\}$$

A. W. Steiner, J. M. Lattimer & E. F. Brown, ApJ 722, 33 (2010)

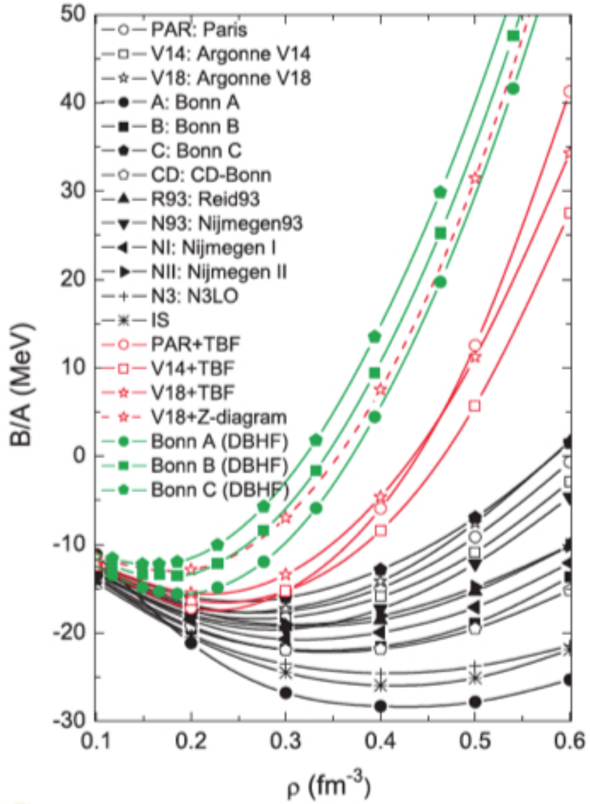
Three-Nucleon Forces

Necessary to:

- Reproduce the spectra of light nuclei
- Saturate properly in non-relativistic many-body calculations



S. Pieper & R.B. Wiringa, Annu. Rev. Nucl. Part. Sci. 51, 53 (2001)



Z. H. Li et al., PRC 74, 047304 (2006)

Exotic Matter:
The Hyperon Puzzle

"Hyperons → "soft (or too soft) EoS" not compatible (mainly in microscopic approaches) with measured (high) masses. However, the presence of hyperons in the NS interior seems to be unavoidable."

- ✓ can YN & YY interactions still solve it ?
- ✓ or perhaps hyperonic three-body forces ?
- ✓ what about quark matter ?

Approaches to the Nuclear EoS

Phenomenological approaches

Based on effective density-dependent interactions with parameters adjusted to reproduce nuclear observables and compact star properties

- ❖ Liquid drop type: BPS, BBP, LS, OFN
- ❖ Thomas-Fermi: Shen
- ❖ ETFSI: BSk
- ❖ HF: NV, Sk, PAL, RMF, RHF, QMC
- ❖ Statistical models: HWN, RG, HS

I apologize for all
those approaches
I have missed

Microscopic ab-initio approaches

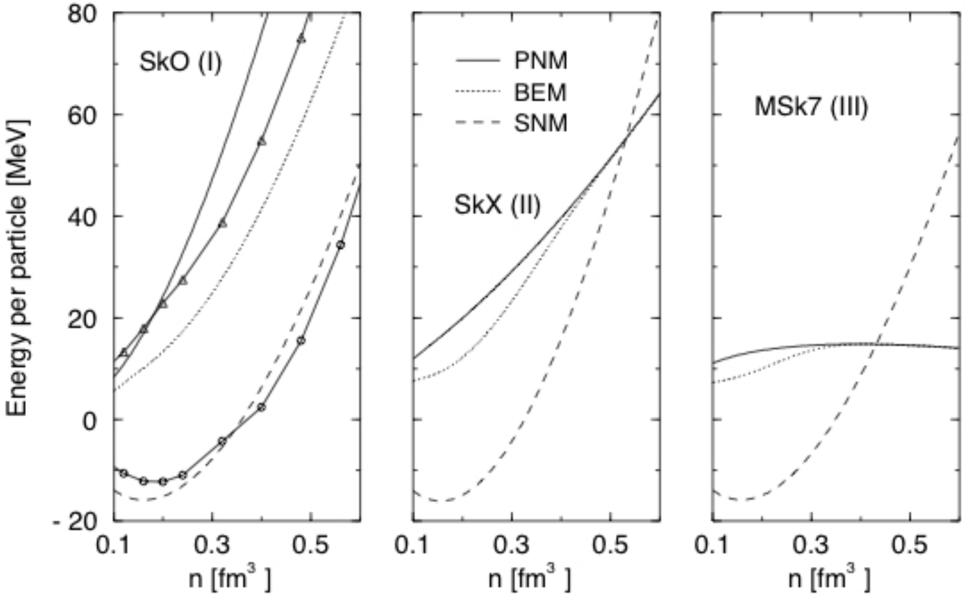
Based on two- & three-body realistic interactions. The EoS is obtained by "solving" the complicated many-body problem

- ❖ Variational: APS, CBF, FHNC, LOVC
- ❖ Monte-Carlo: VMC, DMC, GFMC, AFDMC
- ❖ Diagrammatic: BBG (BHF), SCGF
- ❖ RG methods: $V_{low\ k}$ & SRG from χEFT potentials
- ❖ DBHF

A glance on the phenomenological models

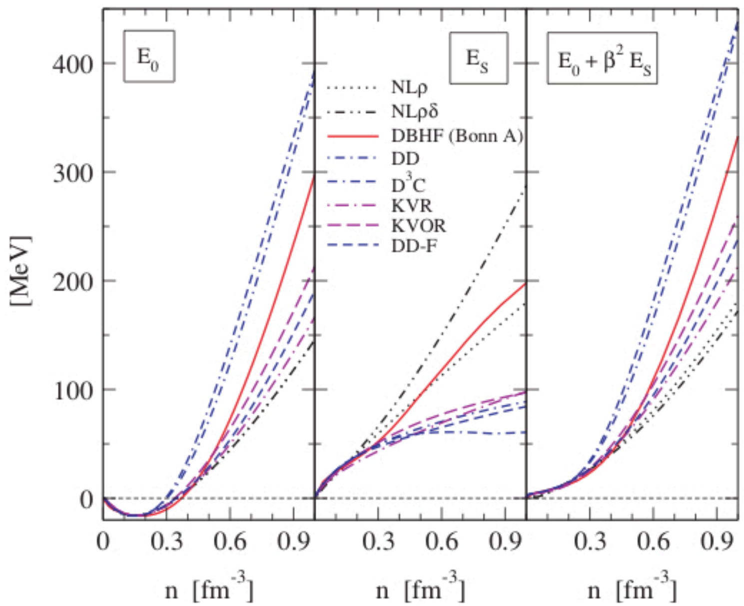
Proliferation of phenomenological models predicting different SM & NM EoS

Skyrme



J. R. Stone et al., PRC 68, 034324 (2003)

RMF



T. Klan et al., PRC 74, 035802 (2006)

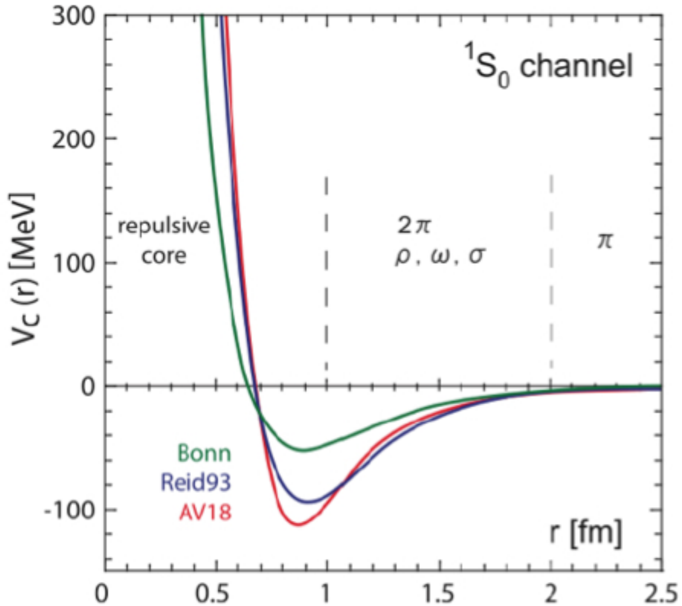
Recently M. Dutra et al., (arXiv:1405.3633) have analyzed 263 parametrizations of 7 different types of RMF imposing constraints from SM, PNM & Symmetry Energy and its derivatives. Similar analysis was done for 240 Skyrme forces by M. Dutra et al., (PRC 85, 035201 (2012))

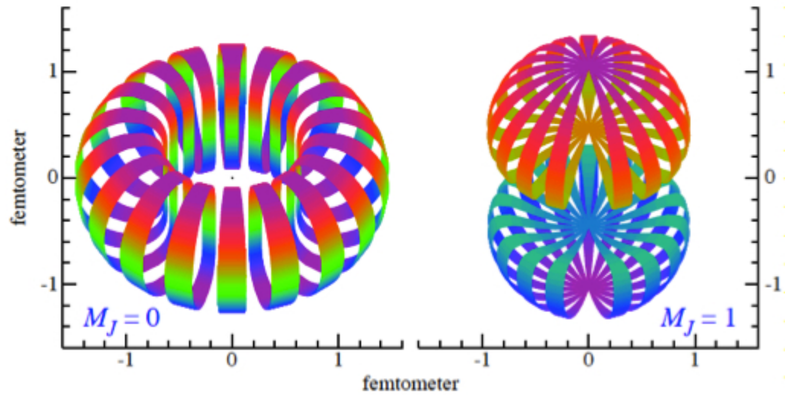
Difficulties of microscopic approaches

- Different NN potentials in the market

- Short range repulsion makes any perturbation expansion in terms of V meaningless. Different ways of treating SRC

- Complicated channel & operatorial structure (central, spin-spin, spin-isospin, tensor, spin-orbit, …)





A comparison of some microscopic approaches

Compare different many-body techniques using the same NN interaction to find the sources of discrepancies & ultimately determine "systematic error" associated to the nuclear EoS predicted predicted by many-body theory

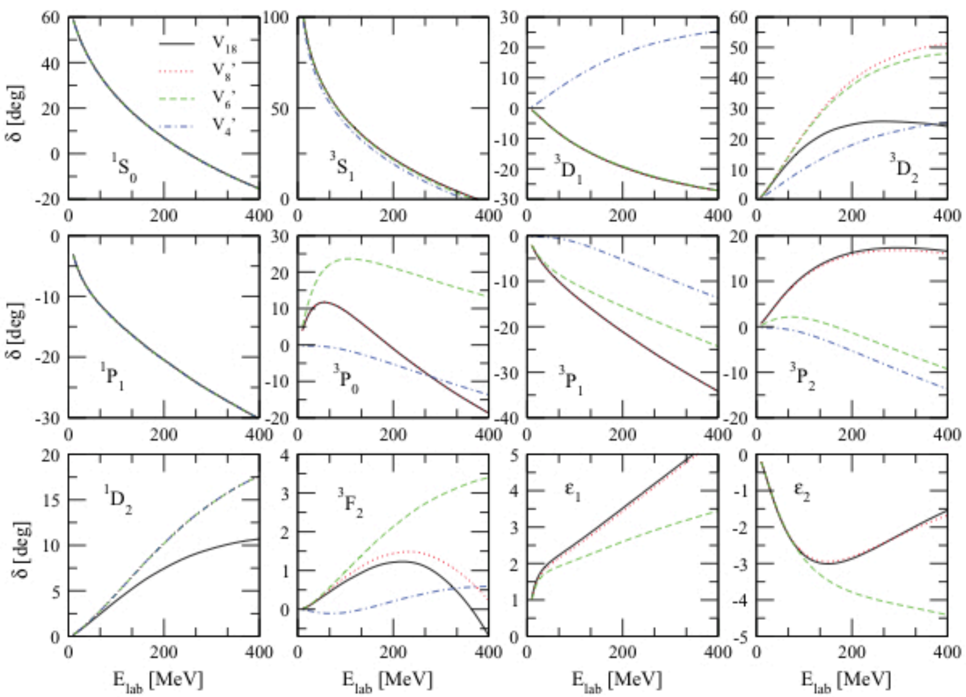
The NN: Av18 & simpler ones: Av4', Av6' & Av8' (3BF are not included)

$$V_{ij} = \sum_{p=1,18} V_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,14} = \left[1, (\vec{\sigma}_i \cdot \vec{\sigma}_j), S_{ij}, \vec{L}\cdot\vec{S}, L^2, L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j), (\vec{L}\cdot\vec{S})^2\right]$$

$$\otimes \left[1, (\vec{\tau}_i \cdot \vec{\tau}_j)\right]$$

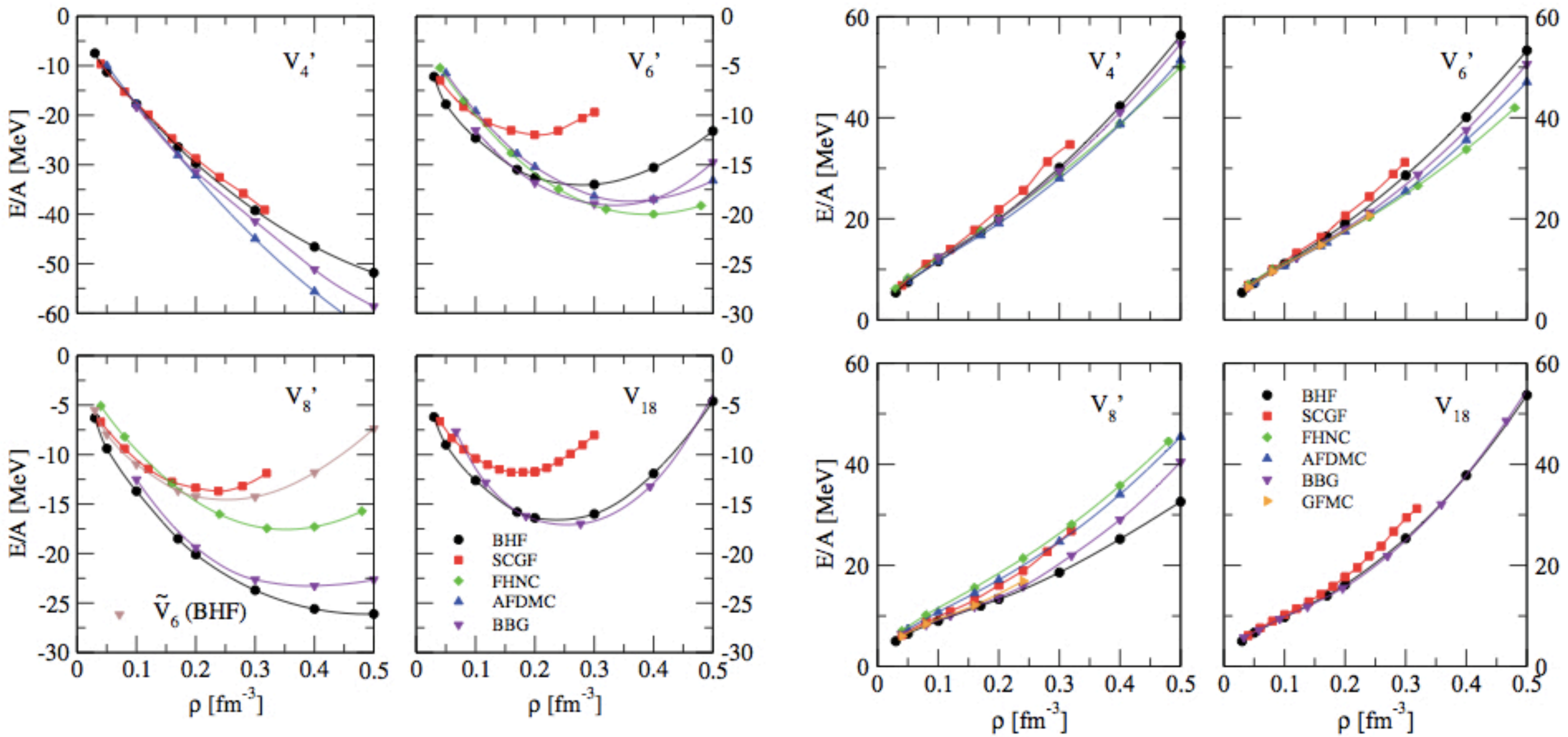
$$O_{ij}^{p=15,18} = \left[T_{ij}, (\vec{\sigma}_i \cdot \vec{\sigma}_j) T_{ij}, S_{ij} T_{ij}, (\tau_{zi} + \tau_{zj})\right]$$



M. Baldo, A. Polls, A. Rios, H.-J. Schulze & I.Vidaña, PRC 86, 064001 (2012)

A comparison of some microscopic approaches

Symmetric nuclear matter　　　　Pure neutron matter



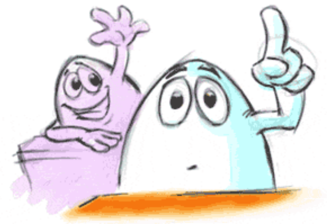
Tensor & spin-orbit and their in-medium treatment are at the heart of most of the observed discrepancies

M. Baldo, A. Polls, A. Rios, H.-J. Schulze & I.Vidaña, PRC 86, 064001 (2012)

A First Attempt of Classification

Approach	Type	β-eq.	T=0	Finite T	Inhom. Matt.	Hom. Matt.	CompOSE	Form
LDM	Phen	X	X	X	X	X	X	Table
TF	Phen	X		X	X			Table
ETFSI	Phen	X	X		X	X		Analytic & Table
HF	Phen	X	X	X				Table
SM	Phen	X	X	X	X	X	X	Table
Var	Micr	X	X			X		Analytic & Table
MC	Micr	X	X			X		Analytic & Table
Diagr	Micr	X	X	X		X		Table
RG	Micr		X			X		Table
DBHF	Micr	X	X			X		Table

Open Questions



- Why is the isovector part of the EoS & particularly the density dependence of the nuclear symmetry energy so uncertain ?

- What is the role of many-body cluster contributions ?

- Is there a solution for the hyperon puzzle ? . If yes, what it is ?
- And many, many others …

Incomplete List of References

<u>Phenomenological approaches</u>

❖ Liquid drop type

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❖ Thomas-Fermi

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❖ ETFSI

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❖ Hartree-Fock

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Microscopic approaches

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- ❖ Monte Carlo

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- ❖ DBHF

  B. Ter Haar & R. Malfiet, Phys, Rep. 149, 207 (1987); Phys. Rev. C 36, 1611 (1987)
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The discussion is served