# Superfluidity and Superconductivity in Compact Stars

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WG meeting - November 18th, 2014

The aim of this talk is to **initiate and motivate discussions** about superfluidity and superconductivity in compact stars.

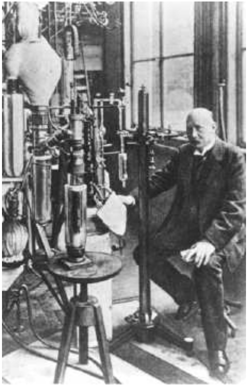
Memorandum of Understanding of the COST Action MP1304:

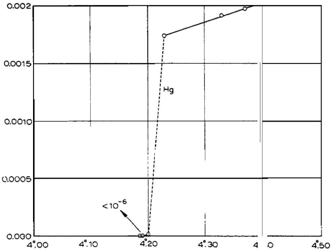
> *"Investigate superfluidity and superconductivity in dense matter. The WG will employ the numerous observations to understand the occurrence of superfluidity and superconductivity in various regions of compact stars and predict signatures revealing their presence in compact stars."*

`http://compstar.uni-frankfurt.de/wp-content/uploads/2014/02/MP1304-e.pdf`

# Discovery of superconductivity

Superconductivity was discovered by Heike Kamerlingh Onnes and his collaborators in 1911 using liquified helium.






But the explanation had to wait for the Bardeen-Cooper-Schrieffer (BCS) theory in 1957.

Onnes also noted "*Just before the lowest temperature was reached, the boiling suddenly stops...*"

About the history of superconductivity:

*van Delft&,Kes, Phys. Today, 63, 9, 38 (2010)*

# Discovery of superfluidity

During the 1930s, it was found by several groups that below $T_\lambda = 2.17$ K, helium does not behave like an ordinary liquid.

"*by analogy with superconductors, the helium below the $\lambda$-point enters a special state which might be called superfluid.*"

*Kapitza, Nature 141, 74 (1938).*

"*the observed type of flow most certainly cannot be treated as laminar or even as ordinary turbulent flow.*"

*Allen and Misener, Nature 141, 75 (1938).*

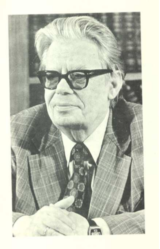

About the history of superfluidity:

*Balibar, J. Low Temp. Phys. 146, 441 (2007).*

# Superfluidity and superconductivity in neutron stars

Before the discovery of pulsars, several superconductors were known but only helium was found to be superfluid.

The BCS theory was applied to nuclei by Bohr, Mottelson, Pines and Belyaev *Phys. Rev. 110, 936 (1958); Mat.-Fys. Medd. K. Dan. Vid. Selsk. 31 , 1 (1959).*



N.N. Bogoliubov, who developed a microscopic theory of superfluidity and superconductivity, was the first to explore its application to nuclear matter.
*Dokl. Ak. nauk SSSR 119, 52 (1958).*

In 1959, Migdal predicted neutron-star superfluidity (*Nucl. Phys. 13, 655*), which was first studied by Ginzburg & Kirzhnits in 1964 (*Zh. Eksp. Teor. Fiz. 47, 2006*).

# Electron superconductivity in neutron stars

**Iron is superconducting** at $\rho \simeq 8.2$ g.cm$^{-3}$ with $T_{ce} \simeq 2$ K, much lower than neutron-star surface temperatures.
*Shimizu et al., Nature 412, 316 (2001)*.

The critical temperature of a uniform non-relativistic electron gas (jelium) is given by ($T_{\rm pi}$ is the plasma temperature)
$T_{ce} = T_{\rm pi} \exp\left(-8\hbar v_{\rm Fe}/\pi e^2\right) \Rightarrow T_{ce} \propto \exp(-\zeta(\rho/\rho_{\rm ord})^{1/3})$ with $\rho_{\rm ord} = m_{\rm u}/(4\pi a_0^3/3)$.

At densities above $\sim 10^6$ g.cm$^{-3}$, electrons become relativistic $v_{\rm Fe} \sim c$ so that ($\alpha = e^2/\hbar c \simeq 1/137$) $T_{ce} = T_{\rm pi} \exp\left(-8/\pi\alpha\right) \sim 0$
*Ginzburg, J. Stat. Phys. 1(1969),3.*

Electrons in neutron stars are not superconducting.

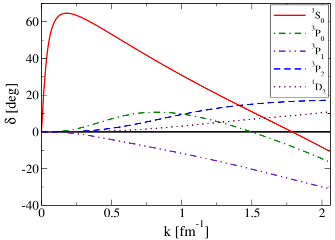
# Nuclear superfluidity and superconductivity

At low enough temperatures, nucleons may form pairs that can condense into a superfluid/superconducting phase.

Most attractive pairing channels ($\delta > 0$):

- $^1S_0$ at low densities
- $^3P_2$ at high densities

*A. Gezerlis, C. J. Pethick, A. Schwenk, arXiv:1406.6109*



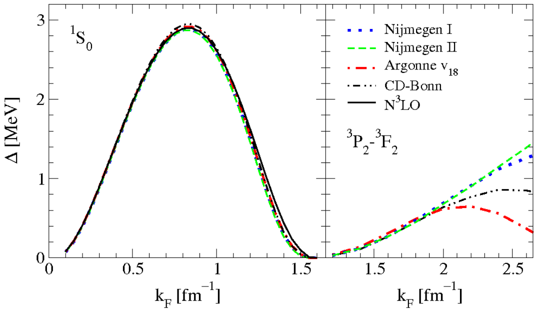

Microscopic calculations of superfluidity/superconductivity in homogeneous nuclear matter:

- diagrammatic methods
- variational methods
- quantum Monte Carlo methods.

Experiments: cold atoms, atomic nuclei

# Pairing in neutron matter: BCS

Lowest order approximation: BCS theory.

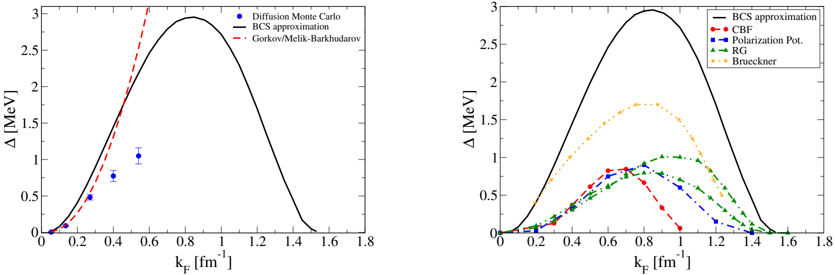



*A. Gezerlis, C. J. Pethick, A. Schwenk, arXiv:1406.6109*

- $^1S_0$ pairing gaps are essentially independent of the NN potential, the role of the NNN potential is negligible.
- $^3P_2$ pairing gaps are very dependent on the NN and NNN potentials.

# $^1S_0$ pairing in neutron matter: beyond BCS

The $^1S_0$ pairing gaps are strongly suppressed by medium effects, and to a lesser extent by NNN forces.



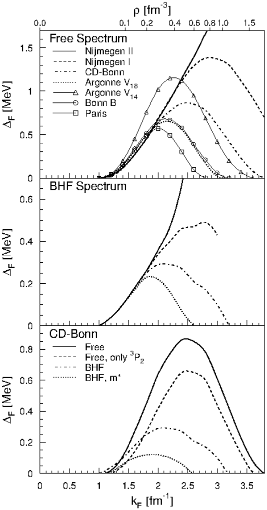

*A. Gezerlis, C. J. Pethick, A. Schwenk, arXiv:1406.6109*

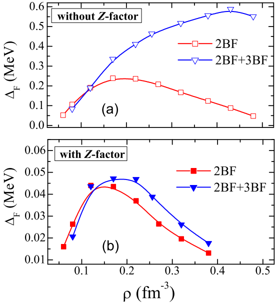
At very low densities, the $^1S_0$ BCS pairing gap is reduced by a factor $(4e)^{-1/3} \simeq 0.45$.

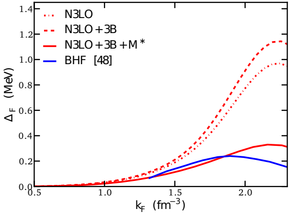
*Gorkov&Melik-Barkhudarov, Sov. Phys. JETP, 13, 1018, (1961).*

# $^3P_2$ pairing in neutron matter: beyond BCS

The $^3P_2$ pairing gaps are also suppressed by medium effects, but are enhanced by NNN forces.




*Maurio et al., PRC90, 044003 (2014)*

*Dong et al., PRC87, 062801(R) (2013)*

*Baldo et al., PRC58, 1921 (1998)*

# Pairing in neutron star cores

The interior of a neutron star is not only made of neutrons, but consists of protons, leptons, hyperons, and possibly mesons, and even deconfined quarks.

Possible phases:

- $^1S_0$ and $^3P_2$ proton pairing
- neutron-proton pairing
- hyperon-hyperon pairing ($^1S_0$ $\Lambda\Lambda$)
- hyperon-nucleon pairing ($^1S_0$ $n\Lambda$, $^1S_0$ $n\Sigma^-$, $^3SD_1$ $n\Sigma^-$)
- quark pairing

Although $^1S_0$ proton superconductivity is well established, the other superfluid/superconducting phases have been much less studied.

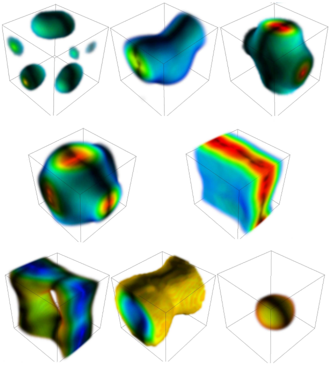
# Pairing in neutron star crusts

The neutron superfluid in the inner crust of a neutron star coexists with an assembly of neutron-proton clusters, which influence superfluidity.

Because of inhomogeneities, microscopic calculations based on realistic forces are not feasible.

Phenomenological approaches:

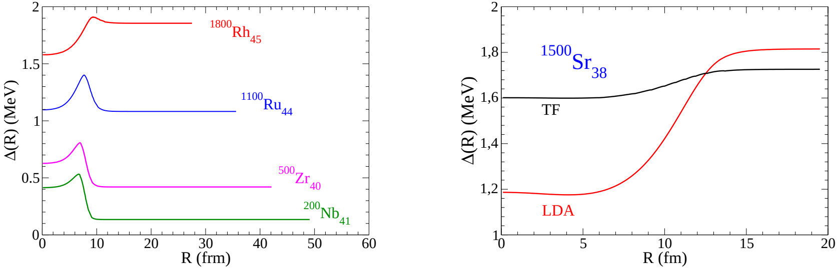
- local density approximation
- semi-classical methods
- self-consistent mean-field methods/density functional theory.
- beyond mean-field methods



*Schuetrumpf et al., PRC87, 055805 (2013)*

# Pairing in neutron star crusts

Semi-classical methods:



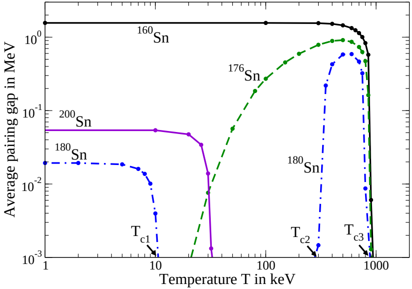

*Schuck & Vinas, in "50 years of nuclear BCS" (World Scientific Publishing, 2013), pp. 212-226*

Superfluidity is **highly non-local** due to proximity effects (the coherence length is large compared to spatial density fluctuations). But quantum effects are not fully taken into account.

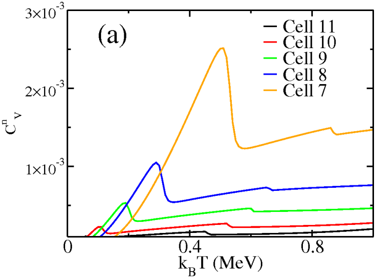
# Pairing in neutron star crusts

BCS or Hartree-Fock-Bogoliubov methods:

*Margueron & Sandulescu, in "Neutron Star Crust" (Nova Science Publisher, 2012).*




*Margueron & Khan, PRC86, 065801 (2012).*



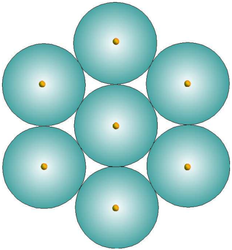

*Pastore, PRC86, 065802 (2012)*

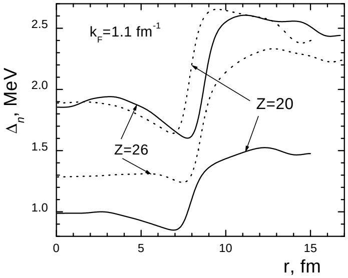
Superfluidity in the crust is very different from neutron matter:

- reentrance phenomenon
- existence of several critical temperatures.

# Pairing in neutron star crusts

The Wigner-Seitz approximation allows for fast numerical computations, but becomes unreliable in the deep regions of the crust (nonuniqueness of boundary conditions).






*Baldo et al., Eur.Phys.J. A32, 97(2007).*

These limitations can be circumvented by the use of the band theory, but so far restricted to BCS.

*Chamel et al., PRC81,045804(2010).*

# Collective excitations

Low energy collective excitations are important for determining transport properties.

neutron matter:

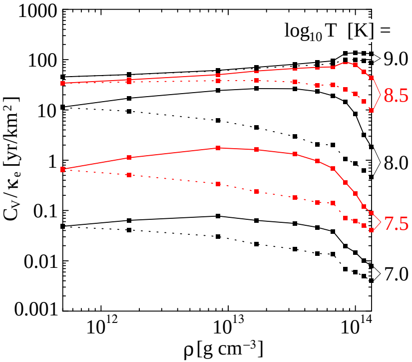
- Landau approximation
- QRPA

neutron-star crusts:

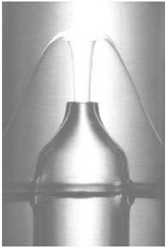
- Hydrodynamic approach
- QRPA + W-S approx.

Due to entrainment, the Bogoliubov-Anderson phonons are strongly mixed with longitudinal lattice phonons: the specific heat is enhanced while the thermal conductivity is reduced.

*Chamel,Page,Reddy,PRC87,035803(2013).*



# Superfluids are multi-fluid systems



J.F. Allen and H. Jones, discovered the **fountain effect**: when heat is supplied on one side of a porous plug, the pressure of superfluid helium increases so much that it produces a liquid jet.

*Allen & Jones, Nature 141, 243 (1938).*

L. Tisza and L. Landau showed that these phenomena can be explained by a **two-fluid model**.

**Relativistic multifluid hydrodynamics** is required for neutron stars:

*Andersson & Comer, Living Rev. Relativity 10 (2007), 1.*

*Carter, in "Physics of Neutron Star Interiors", Lecture Notes in Physics 578, p.54 (2001).*

# Mutual entrainment

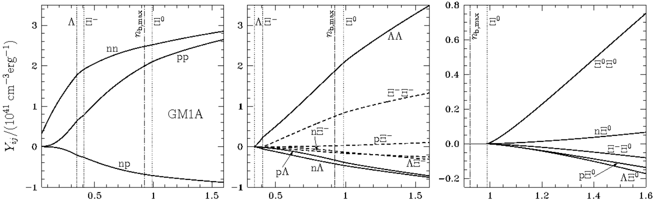
In superfluid mixtures such as $^3$He-$^4$He, the different superfluid constituents may still be **mutually coupled**.

*Andreev & Bashkin, Sov. Phys. JETP 42, 164 (1975).*

Generalized equations of state are needed for modelling superfluid neutron stars.

Microscopic calculations of entrainment matrix:

- (non)relativistic Fermi liquid theory
- (non)relativistic mean field theory.

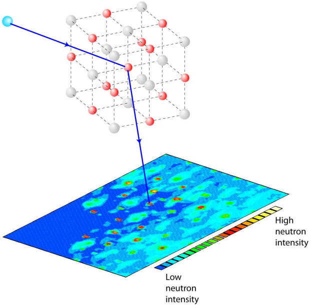



*Gusakov, Haensel, Kantor, MNRAS 439, 318 (2014).*

# Superfluid hydrodynamics in neutron-star crusts

Despite the absence of viscous drag, the crust can still resist the flow of the superfluid due to Bragg scattering.

*Chamel,PRC85,035801(2012).*




Superfluid hydrodynamics in magneto-elastic crust:

- non-relativistic formulation

  *Pethick, Chamel & Reddy, Prog. Theo. Phys. Sup. 186, 9 (2010).*

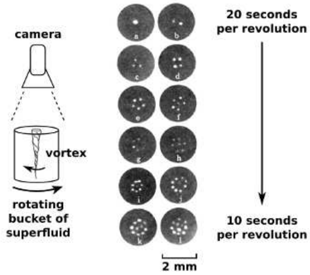
  *Carter & Chachoua, IJMP D15, 1329 (2006).*

- relativistic formulation

  *Carter & Samuelsson, Class.Quant. Grav.23, 5367 (2006).*

# Superfluid vortices

A rotating superfluid is threaded by **quantized vortex lines**, each of which carries an angular momentum $\hbar$. Their surface density of vortices is given by $n_v(\mathrm{km}^{-2}) \sim 10^{14}/P(\mathrm{s})$.




*Yarmchuk, Gordon, and Packard, PRL43, 214 (1979).*

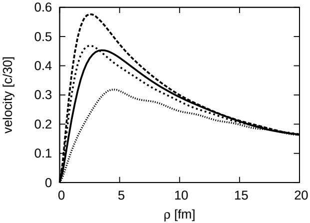
Likewise a type II superconductor is threaded by **flux tubes**.

In a neutron star, the surface density of (proton) flux tubes is expected to be typically $\sim 10^{13} - 10^{14}$ times larger than that of (neutron) vortices. But type I superconductivity is not excluded.
*Sedrakian & Clark, in "Pairing in Fermionic Systems", (World Scientific, 2006)*

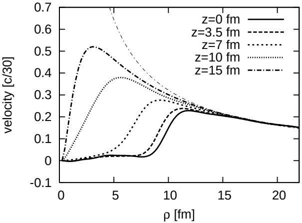
# Superfluid vortices in neutron star crust

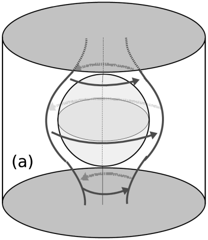
Neutron superfluid vortices can pin to clusters.

single vortex




single vortex + cluster




*Avogadro et al, Nucl.Phys.A811,378(2008).*

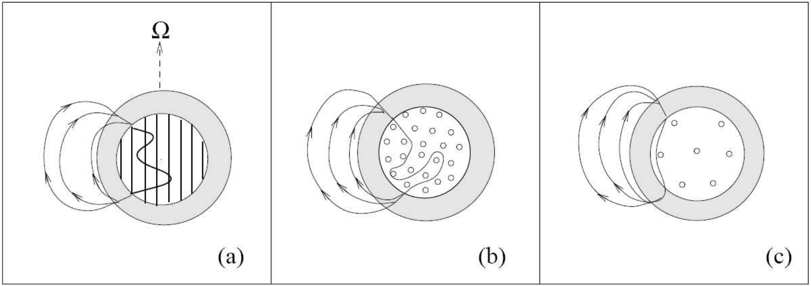
Microscopic calculations of pinning forces:

- local density approximation
- semi-classical methods
- self-consistent mean-field methods

The actual pinning of vortices depends also on the structure of the crust, on the rigidity of the lines and on the vortex dynamics.
*Bulgac, Forbes, Sharma, PRL110, 241102 (2013)*

# Superfluid vortices in neutron star core

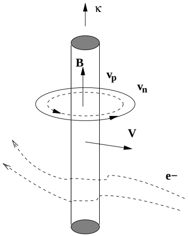
Neutron superfluid vortices can also pin to flux tubes provided the superconductor is of type II. The strong interaction between neutron superfluid vortices and proton flux tubes in neutron star cores leads to movement of crustal plates.




The evolution of the pulsar spin and magnetic field are intimately related to superfluidity and superconductivity

*Ruderman, Astrophys. Space Sci.357, 353 (2009)*

# Superfluid vortices in neutron star core




*picture from K. Glampedakis*

Due to entrainment, neutron vortices carry a fractional quantum flux. Electrons scattering off the magnetic field leads to a **mutual friction** force which could be an important dissipative mechanism for neutron star oscillations.

*Sedrakyan & Shakhabasyan, Astrofizika 8 (1972), 557*

*Sedrakyan & Shakhabasyan, Astrofizika 16 (1980), 727*

*Alpar, Langer, Sauls, ApJ282 (1984) 533*

Superfluid-superconducting hydrodynamics:

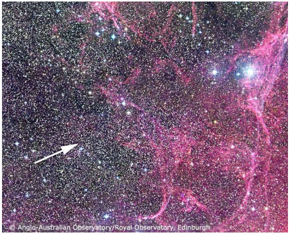
- nonrelativistic formulation

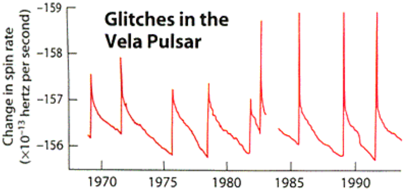
  *Glampedakis,Andersson,Samuelsson, MNRAS410,805(2011)*
- relativistic formulation

  *Carter, in "Vortices in Unconventional Superconductors and Superfluids" (Springer, 2002)*

# Pulsar glitches

The strongest evidence for nuclear superfluidity come from pulsar **sudden spin-ups** since similar phenomena have been observed in laboratory superfluid helium.




So far 451 glitches have been detected in 158 pulsars.

`http://www.jb.man.ac.uk/pulsar/glitches/gTable.html`

Glitches are thought to arise from the unpinning of quantized vortices in neutron-star crust.

*Pines & Alpar, Nature 316, 27(1985)*

# Puzzling glitches

However, this theory has been challenged by observations:

- a huge glitch in PSR 2334+61 *Alpar, AIP Conf.Proc.1379,166(2011)*
- unusual post-glitch relaxation in PSR J1119−6127

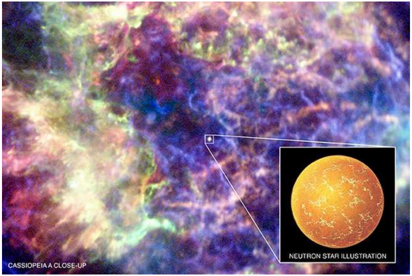
  *Weltevrede et al., MNRAS 411,1917(2011)*
- a huge glitch in PSR J1718−3718 *Manchester & Hobbs, ApJ 736,L31(2011)*
- an anti-glitch in 1E 2259+586 *Archibald et al.,Nature 497,591 (2013).*

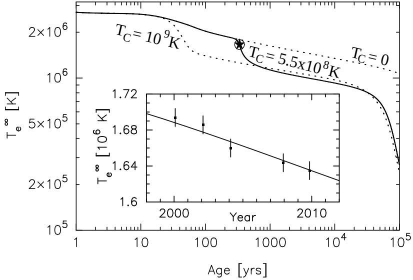
Several theoretical aspects are not well-understood:

- type of superconductivity
- vortex pinning
- superfluid turbulence
- entrainment
- crust-core coupling.

# Cooling of isolated neutron stars

The surface temperature of a neutron star depends on its mass and composition but also on superfluidity. Superfluidity reduces the heat capacity and neutrino emissivities but also opens new channels of neutrino emission.



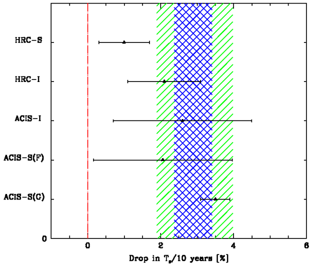

Recent observations of Cassiopeia A provide strong evidence for neutron-star core superfluidity.

*Page et al., PRL 106, 081101; Shternin et al.,MNRAS 412, L108.*

# Cooling of isolated neutron stars

However, a more recent analysis of observational data from all detectors suggests that the cooling rate might be slower.

*Elshamouty et al., ApJ 777, 22 (2013).*




Moreover, alternative scenarios have been proposed.

*Blaschke, Grigorian, Voskresensky, PRC88, 065805 (2013)*

*Bonanno, Baldo, Burgio, Urpin, A&A 561, L5 (2014)*

*Negreiros, Schramm, Weber, Phys. Lett. B718, 1176 (2013)*

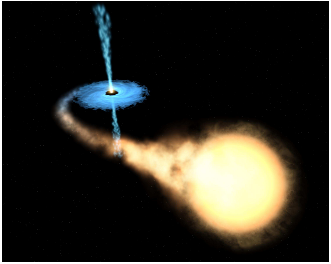
*Sedrakian, A& A 555, L10 (2013)*

Still, most scenarios require superfluidity and/or superconductivity in neutron stars.

# X-ray binaries

Neutron stars in X-ray binaries may be **heated as a result of the accretion of matter** from the companion star.

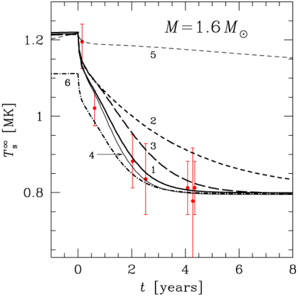
The accretion of matter onto the surface of the neutron star triggers thermonuclear fusion reactions which can become explosive, giving rise to **X-ray bursts**.



In soft X-ray transients (SXT), accretion outbursts are followed by **long period of quiescence** during which the accretion rate is much lower. In some cases, the period of accretion can last long enough for the crust to be **heated out of equilibrium with the core**.

# Thermal relaxation of soft x-ray transients

The thermal relaxation during the quiescent state has been recently monitored for a few accreting neutron stars.




Example: KS 1731−260

Curves 1,3,4 : crystalline crust with neutron superfluidity
Curve 2 : crystalline crust without neutron superfluidity
Curve 5 : amorphous crust with neutron superfluidity

*Shternin et al., Mon. Not. R. Astron. Soc.382(2007), L43.*

*Brown and Cumming, ApJ698 (2009), 1020.*

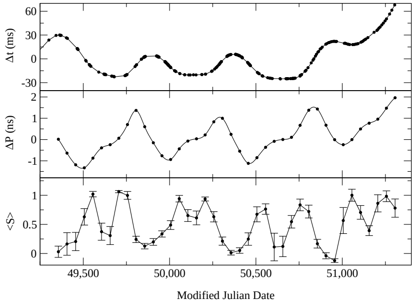
Observations of SXT provide evidence of superfluidity in the inner crust of a neutron star.

# Neutron star precession

**Long-term cyclical variations** of order months to years have been reported in a few neutron stars: Her X-1 (accreting neutron star), the Crab pulsar, PSR 1828−11, PSR B1642−03, PSR B0959−54 and RX J0720.4−3125.

Example: Time of arrival residuals, period residuals, and shape parameter for PSR 1828−11

*Stairs et al., Nature 406(2000),484.*




These variations have been interpreted as the signature of **neutron star precession**.

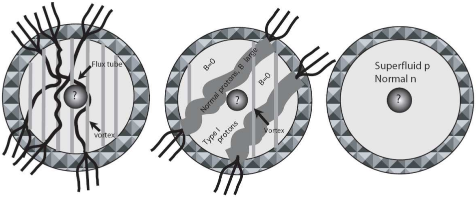
# Precession and superfluidity

For a non-superfluid star with deformation $\epsilon = \Delta I / I$,

$$P_{\text{prec}} = \frac{P}{\epsilon} \gg P$$

For a superfluid star with pinned vortices

$$P_{\text{prec}} = \frac{I_{\text{pin}}}{I} P \ll P$$




*Link, Astrophys. Space Sci.308,435 (2007)*

Observations of precession could thus shed light on superfluidity. On the other hand, precession may trigger instabilities that could unpin vortices.

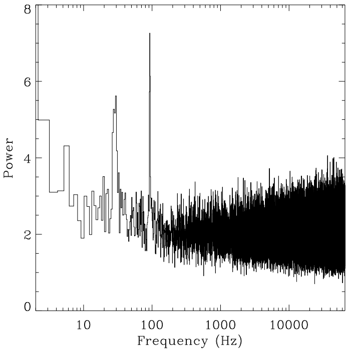
*Glampedakis,Andersson,Jones,PRL100,081101(2008).*

# Asteroseismology of neutron stars

Quasiperiodic oscillations (QPOs) have been detected in the X-ray flux of giant flares from a few soft gamma-ray repeaters.

Example: SGR 1806−20

*Strohmayer&Watts, ApJ653,593 (2006)*




These QPOs are thought to be the signatures of **superfluid magneto-elastic oscillations**.

*Gabler al., PRL 111, 211102 (2013).*

# Discussion: open issues

**Theory**

- Pairing gaps (role of medium effects? inhomogeneities? magnetic field?)
- Collective excitations (spectrum? dispersion relation? impact on transport properties?)
- Entrainment effects
- Type of superconductivity
- Dynamics (vortex pinning? turbulence?)

**Observations**

- Pulsar glitches (mechanism? origin? post-glitch relaxation?)
- Pulsar free precession (pinning? superfluid instabilities?)
- Neutron star oscillations (mode spectrum? role of entrainment? crust-core coupling? damping?)
- Neutron star cooling

**Experiments**

- Hydrodynamics of cold atoms (unitary regime)
- Pairing in finite (hyper)nuclei (binding energies, excitation spectra, 2-nucleon transfer reactions)

# Bibliography

List of <u>some</u> references about superfluidity and superconductivity in compact stars:


**Books/Lecture notes:**

- *Novel Superfluids*, ed. Bennemann&Ketterson (Oxford University Press)
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**Reviews:**

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- Alford, Schmitt, Rajagopal, Schäfer, Rev. Mod. Phys.80, 1455 (2008)
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