

Transport phenomena in supernova and neutron stars

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The aim of this talk is to initiate and motivate discussions about transport phenomena in supernova and neutron stars

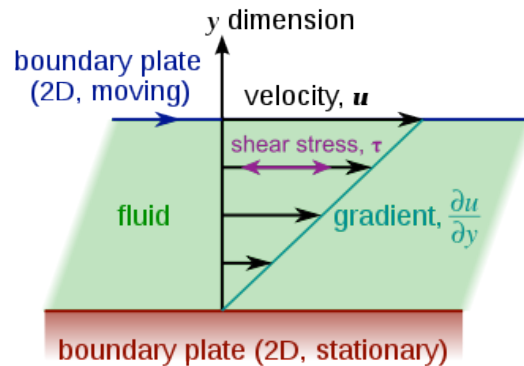
Memorandum of Understanding of the COST Action MP1304:

“Calculate the transport properties and the reaction rates for validated equations of state. The WG will supply the simulations modeling core-collapse supernovae and binary neutron-star mergers with the transport properties and reaction rates of hot and dense nuclear matter”

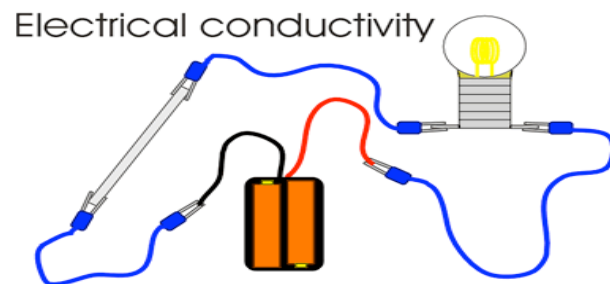
<http://compstar.uni-frankfurt.de/wp-content/uploads/2014/02/MP1304-e.pdf>

Transport coefficients and neutrino transport

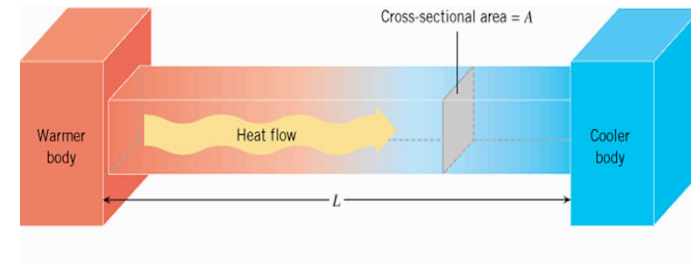
(Shear and Bulk) viscosity:
resistance to gradual deformation
by shear stress or tensile stress



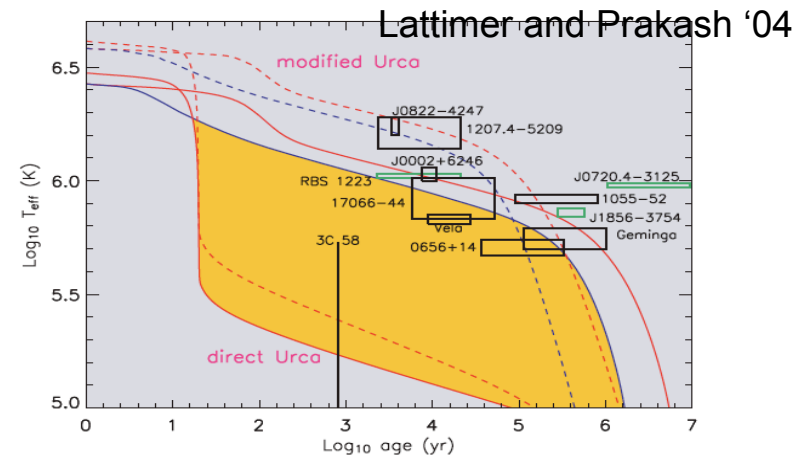
Electrical conductivity:
measures a material's ability
to conduct an electric current



Thermal conductivity:
property of a material to conduct heat



Neutrino transport
by weak decay processes



CRUST

Chamel & Haensel '08

Page & Reddy '12

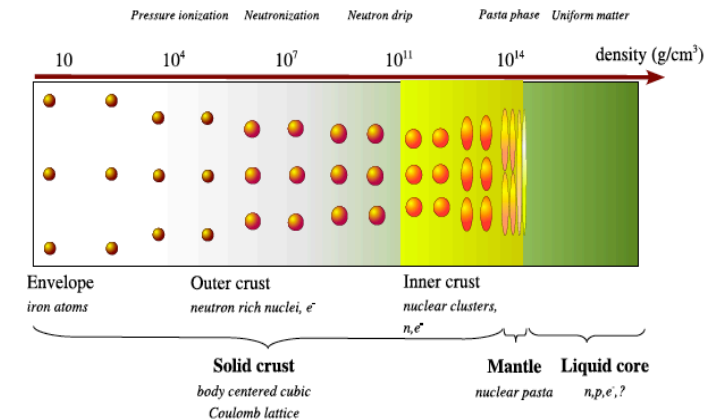
Composed of matter at sub-nuclear density ($\rho \leq 10^{14} \text{ g/cm}^3$) occupying the outermost 1-2 km region of the star

Outer crust: ionized nuclei in a degenerate electron gas

Inner crust: neutrons drip out from nucleus.

In shallower regions, spherical neutron-rich nuclei are embedded in a neutron superfluid. In deeper regions, non-spherical (“pasta”) phases appear

cold non-accreting neutron star:
ground state structure



Generalities:

- **Electrons are strongly degenerate** forming a dilute quasi-ideal gas of “electron excitations”
- **Electrons interact with nuclei** (equivalent to the scattering of electrons with a dilute gas of phonons), **electrons and impurities in the crystal lattice**
- The “**screened**” **Coulomb interaction** can be treated as small perturbation
- Analysis via the **Boltzmann equation**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = I_e[f]$$

where $I_e[f] = I_{eN}[f] + I_{ee}[f] + I_{imp}[f]$, being the scatterers uncorrelated

- First approximation: **relaxation time approximation**

$$I_{eN}[f] \approx -\frac{\delta f}{\tau_0(\epsilon)}$$

- Second approximation: **Matthiessen rule** (sum of frequencies of different scattering processes)

Thermal (κ) and electrical (σ) conductivities

$$\mathbf{j}_T = -Q_T T \mathbf{j}_e - \kappa \nabla T$$

$$\mathbf{j}_e = \sigma \mathbf{E}^* + \sigma Q_T \nabla T,$$

$$\sigma = \frac{e^2 n_e}{m_e^* \nu_\sigma} \quad \nu_\kappa = \nu_{eN}^\kappa + \nu_{ee}^\kappa + \nu_{\text{imp}}^\kappa$$

$$\kappa = \frac{\pi^2 k_B^2 T n_e}{3 m_e^* \nu_\kappa} \quad \nu_\sigma = \nu_{eN}^\sigma + \nu_{\text{imp}}^\sigma$$

Outer crust: main carriers of transport are electrons scattering with ions

$$\nu_\kappa = \nu_\sigma \simeq \nu_{eN}(\mu_e)$$

For low temperatures, impurities dominate

Inner crust: the neutron gas plays a role

$$\kappa = \kappa_e + \kappa_n$$

with κ_n ($\kappa_n^{\text{diff}}, \kappa_n^{\text{conv}}$ in superfluids)

$$\kappa_n^{\text{diff}} = \frac{\pi^2 k_B^2 T n_n}{3 m_n^* \nu_n} \quad \nu_n = \nu_{nN} + \nu_{nn} \quad \nu_n \approx \nu_{nN}$$

Electron-electron might also be more important than electron-ion interactions (Shternin & Yakovlev '06)

Chamel and Haensel '08

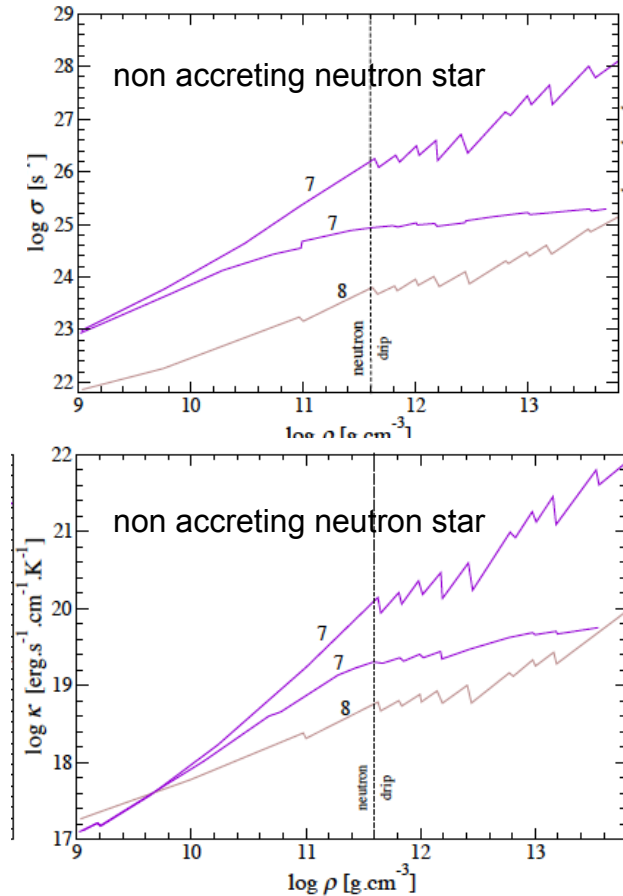


Figure 54: Electrical conductivity σ and thermal conductivity κ of the outer and inner crust, calculated for the ground-state model of Negele & Vautherin [303]. Labels 7 and 8 refer to $\log_{10} T [\text{K}] = 7$ and 8, respectively. The thin line with label 7 corresponds to an impure crust, which contains in the lattice sites 5% impurities – nuclei with $|Z_{\text{imp}} - Z| = 4$. Based on a figure made by A.Y. Potekhin.

Viscosities

$$\Pi_{ij}^{\text{vis}} = \eta \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{U} \right) + \zeta \delta_{ij} \nabla \cdot \mathbf{U}$$

Shear viscosity η

For strongly non-ideal and solid plasma, the transport is mediated by electrons

Outer crust:

$$\eta = \eta_e + \eta_N$$

$$\rho > 10^5 \text{ g cm}^{-3} \quad \eta_e \gg \eta_N \quad \text{and} \quad \eta \approx \eta_e$$

Electrons scatter on nuclei, on impurity nuclei and themselves

$$\nu_e^\eta = \nu_{eN}^\eta + \nu_{\text{imp}}^\eta + \nu_{ee}^\eta$$

For not too low T , $\nu_e \approx \nu_{eN}$

At low temperatures, impurities dominate

Inner crust: extra contribution of gas of neutrons

Bulk viscosity ξ

calculations? It is assumed that $\zeta \ll \eta$.

Chamel and Haensel '08

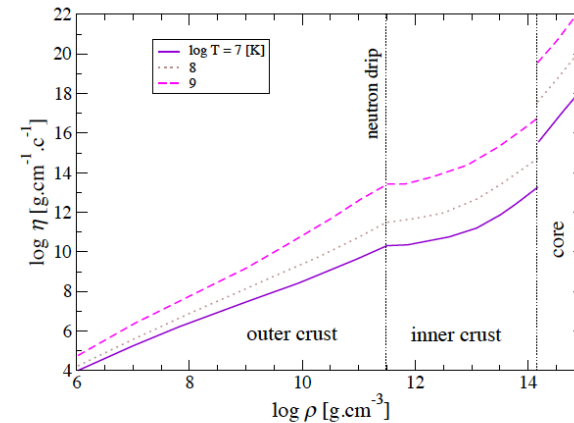


Figure 56: Electron shear viscosity of the crust and the upper layer of the core for $\log_{10} T[\text{K}] = 7, 8, 9$. Calculated by Chugunov & Yakovlev [101] with a smooth composition model of the ground-state (Appendix B of Haensel, Potekhin, and Yakovlev [184]).

Remarks on inner crust

Page & Reddy '12

dynamics is dominated by electrons and lattice phonons (coherent motion of the clusters of protons located in neutron-rich nuclei). Another ingredient is superfluid phonons.

Conductivity and viscosity

- electron Umklapp scattering off the ion lattice is efficient for typical temperatures of interest
- at low temperature when Umklapp scattering is suppressed, electron-impurity scattering will likely dominate

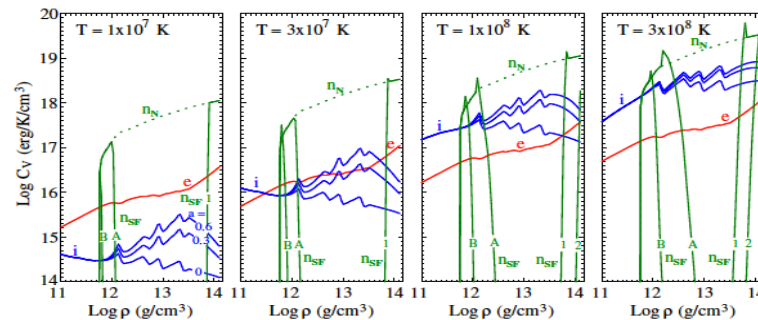
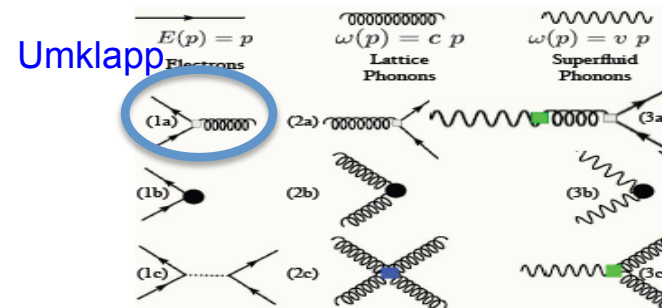
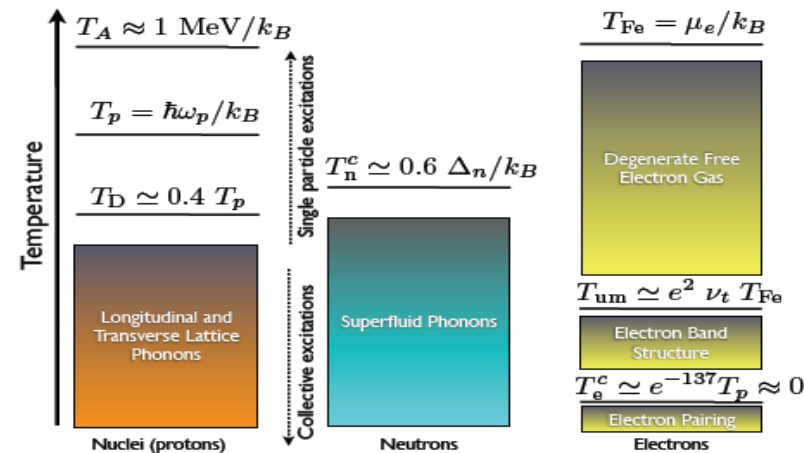


Figure 5. Specific heat of ions, electrons, and for neutrons with (labelled n_{SF}) and without the effects of the superfluid gap (labelled n_N) are shown for four representative temperatures.

Specific heat (thermal property)
ion, electron and neutron contributions

Strong magnetic fields

Overview of Chamel & Haensel '08
based on Potekhin '99; Ventura & Potekhin '01

strong surface magnetic field: $B \gg 10^9$ G
electron transport processes are affected

transport properties become anisotropic:
magnetic field bends electron trajectories in the (x,y)
plane and suppresses the electron transport across B

Different scenarios:

Nonquantizing magnetic fields:

many Landau orbitals are populated and
quantum effects smeared by thermal effects

Weakly-quantizing magnetic fields

- many Landau orbitals are populated and quantum effects are well pronounced
(oscillations due to filling of a new Landau level)

-two relaxation times, parallel and perpendicular to B field

Strongly-quantizing magnetic fields

quantization effects are well pronounced and electrons populate the ground
Landau level

Some conclusions:

along B, thermal electron conduction is bigger than thermal ion conduction
across B, thermal electron conduction is suppressed

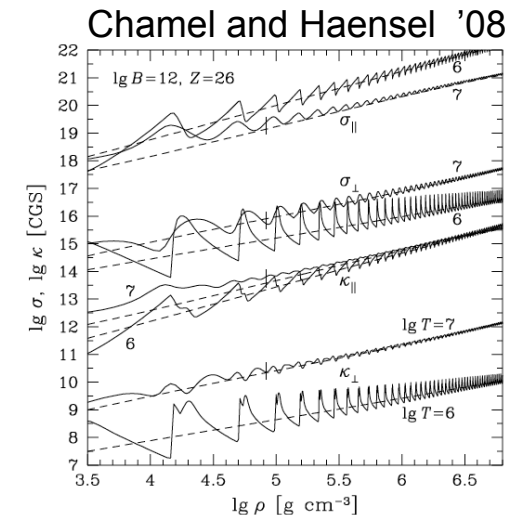


Figure 59: Longitudinal (\parallel) and transverse (\perp) electrical and thermal conductivities in the outer envelope composed of ^{56}Fe for $B = 10^{12}$ G and $\log_{10} T [\text{K}] = 6, 7$. Quantum calculations (solid lines) are compared with classical ones (dash lines). Vertical bars: liquid-solid transition at $T = 10^7$ K. Based on Figure 5 from [338].

Possible observations to constrain transport properties of the crust

- **supernova** observations:
the constituents of the collapsing stellar cores and neutron star crusts are the same
- **cooling of a young neutron star** is very sensitive to its crust physics:
thermal relaxation of the crust depends on specific heat and thermal conductivity
- **r-mode damping** due to viscous crust-core Ekman layer
- relaxation of the crust after deposit of heat in
 - accretion in a binary system, such as **Quasi-Persistent Sources of SXRTs**
 - **giant flares**

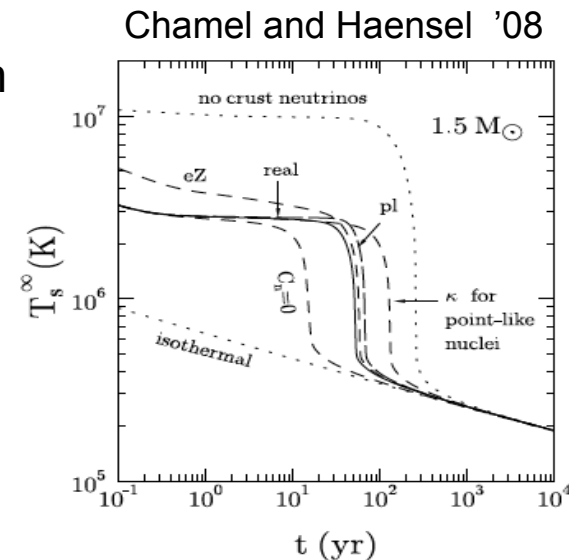


Figure 70: Effective surface temperature (as seen by an observer at infinity) of a $1.5 M_\odot$ neutron star during the first hundred years for different crust models. Dotted lines: cooling without neutrino emission from the crust (upper line), infinite κ at $\rho > 10^{10} \text{ g cm}^{-3}$. Solid line: cooling curve for the best values of κ , C_v , and Q_ν . Dashed line $C_n = 0$: dripped neutrons heat capacity removed. Dashed curve κ : thermal conductivity calculated assuming point-like nuclei. Two other dashed lines: neutrino emission processes removed except for plasmon decay (pl) or electron-nucleus Bremsstrahlung (eZ). See also line $1.5 M_\odot$ in Table 2 of [167].

CORE

Outer core extends in a density range $0.5 \rho_0 \leq \rho \leq 2\rho_0$ and can be several kilometers deep. It is mainly composed of neutrons with some admixture of protons, electrons and muons. Neutron superfluidity and proton superconductivity are expected.

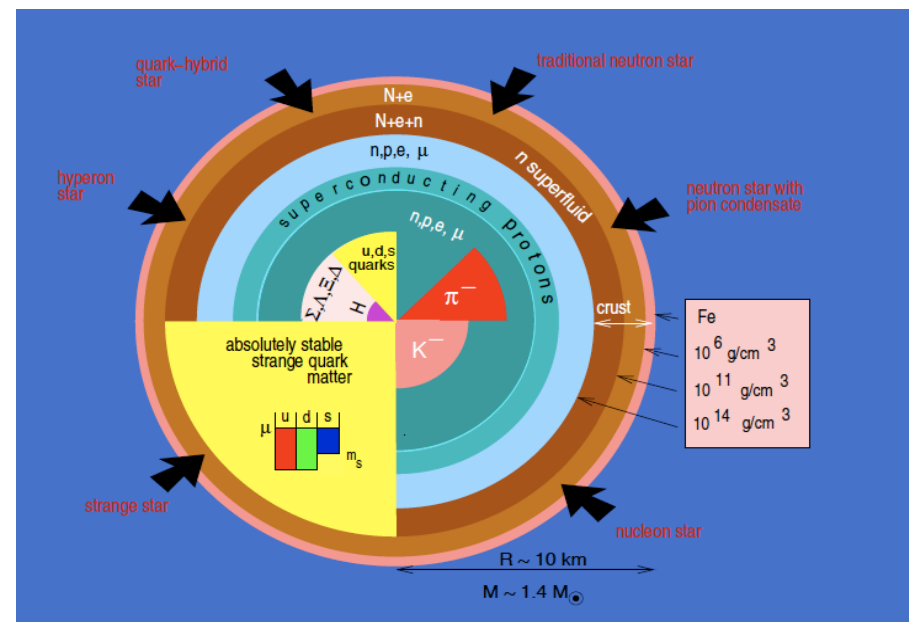
Inner core can be several kilometers in radius and have a central density as high as $10\text{--}15 \rho_0$. The composition and equation of state of the inner core are poorly known.

Studied scenarios:

Matter made of n, p, e, μ

Matter made of n, p, e, μ, Y (=hyperons)

Matter made of quarks



Fridolin Weber

n,p,e, μ matter

electrons and muons (lightest and most mobile particles) and neutrons (most abundant)

Shear viscosity $\eta = \eta_{e\mu} + \eta_n$

$e\mu$ collisions with protons:

Flowers and Itoh '76 '79; Cutler & Lindblom '87
Shternin and Yakovlev '08

n-n and n-p collisions:

Flowers and Itoh '79, Cutler and Lindblom '87,
Benhar and Valli '07, Shternin and Yakovlev '08..

superfluidity:

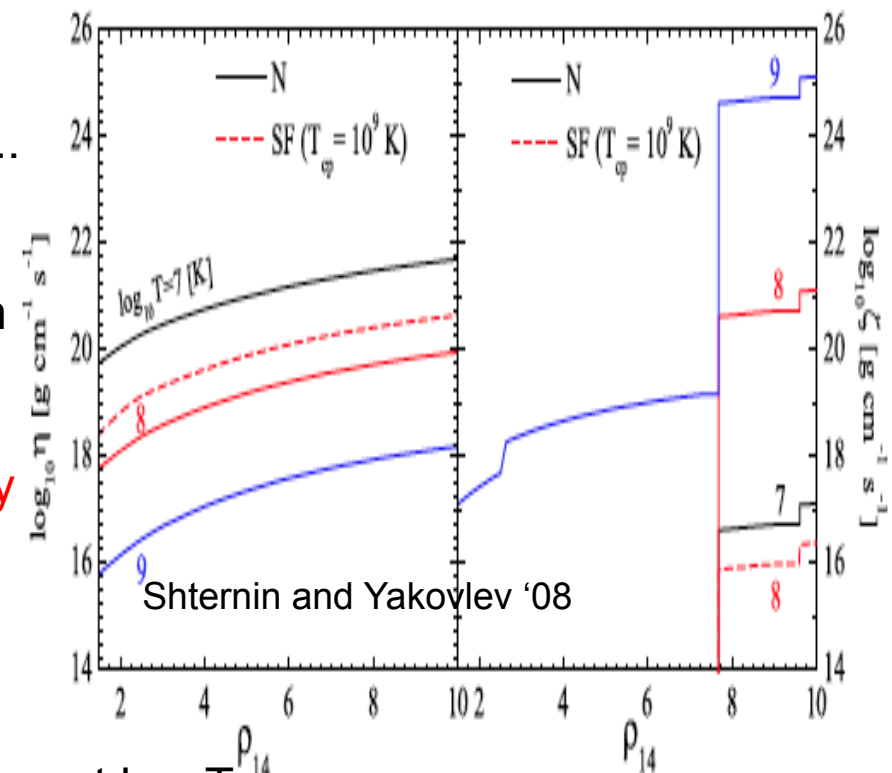
-proton superconductivity affects $e\mu$ interaction
-neutron superfluidity, phonon-phonon
contribution (Manuel and LT '11,'13) and/or
electron-phonon contribution (Bedaque & Reddy
'13, Reddy et al'14)

Conclusions:

- $\eta_{e\mu}$ generally dominates over η_n
- η is comparable with ξ at $T \sim 10^8$ K and dominates at low T
- proton superconductivity increases the importance of η in comparison with ξ
- neutron superfluidity: phonon-phonon/electron contributions might become important

Bulk viscosity

direct and modified Urca processes
in the core of a vibrating neutron star
by Sawyer'89; Haensel & Schaeffer'92;
Haensel, Levenfish & Yakovlev '00 '01



n,p,e, μ matter

Thermal conductivity

$$\kappa \approx \kappa_b + \kappa_{e\mu}$$

κ_b : Baiko, Hensel & Yakovlev '01, ...

$\kappa_{e\mu}$: Flowers & Itoh '76 '79; Gnedin & Yakovlev '95; Shternin & Yakovlev '07

Conclusions:

- $\kappa_{e\mu} \geq \kappa_n$ for $T \geq 2 \times 10^9$ K in normal matter and for any T in superconducting matter with proton critical temperatures $T_{cp} \geq 3 \times 10^9$ K
- in **neutron superfluid matter**:
phonon contribution might be important (Manuel & LT '14) and/or electron-phonon (Bedaque & Reddy '13, Reddy et al'14)

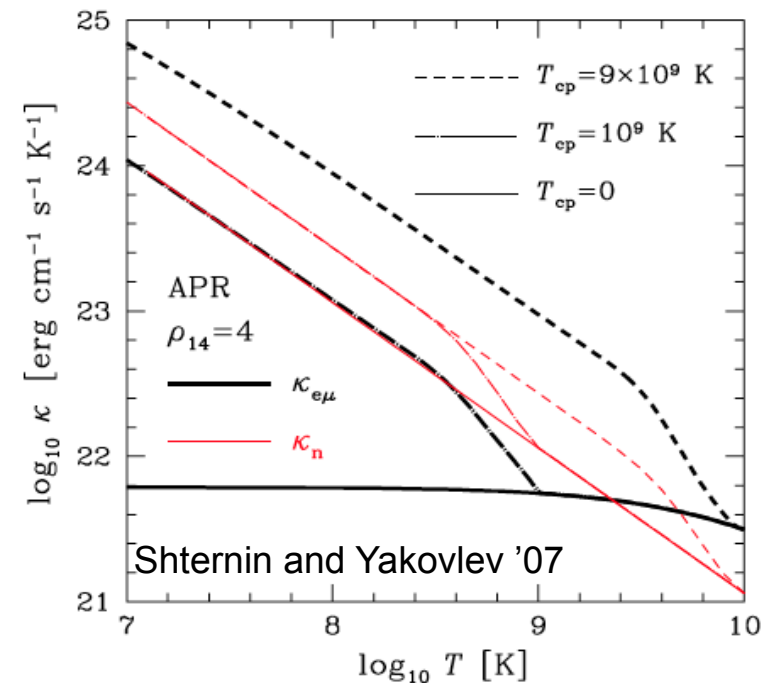
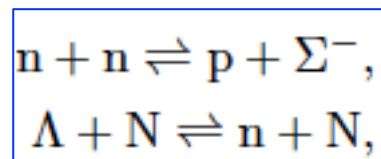


FIG. 5 (color online). Temperature dependence of the electron-muon thermal conductivity $\kappa_{e\mu}$ (thick lines) and the neutron conductivity κ_n (thin lines) in a neutron star core with the APR EOS at $\rho = 4 \times 10^{14}$ g cm⁻³. Solid lines refer to nonsuperconducting matter, while dot-dashed and dashed lines are for matter with proton superconductivity (with $T_{cp} = 10^9$ and 9×10^9 K, respectively).

n,p,e, μ ,Y (=hyperon) matter

Bulk viscosity Jones '71 '01, Lindblom & Owen '02, Haensel, Levenfish & Yakovlev '02, Chatterjee & Bandyopadhyay '07,...

Hyperons: direct non-leptonic hyperon collisions which go via weak interaction such as



are the most efficient mechanism involving hyperons

Only with Σ^-

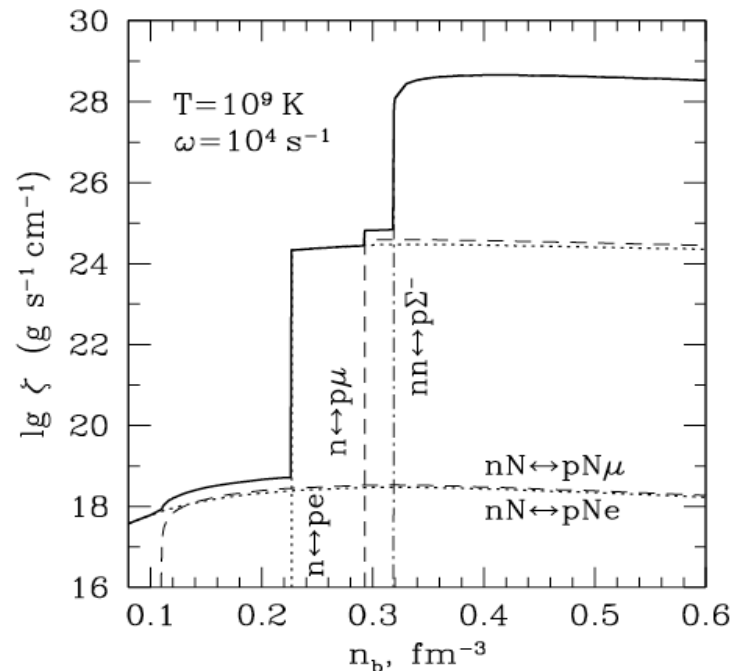


Fig. 1. Density dependence of partial bulk viscosities associated with various processes (indicated near the curves) at $T = 10^9$ K and $\omega = 10^4$ s $^{-1}$ in non-superfluid matter. Dotted and dashed lines refer to Urca processes involving electrons and muons, respectively; dot-and-dashed line refers to hyperon process (3). Thick solid line is the total bulk viscosity.

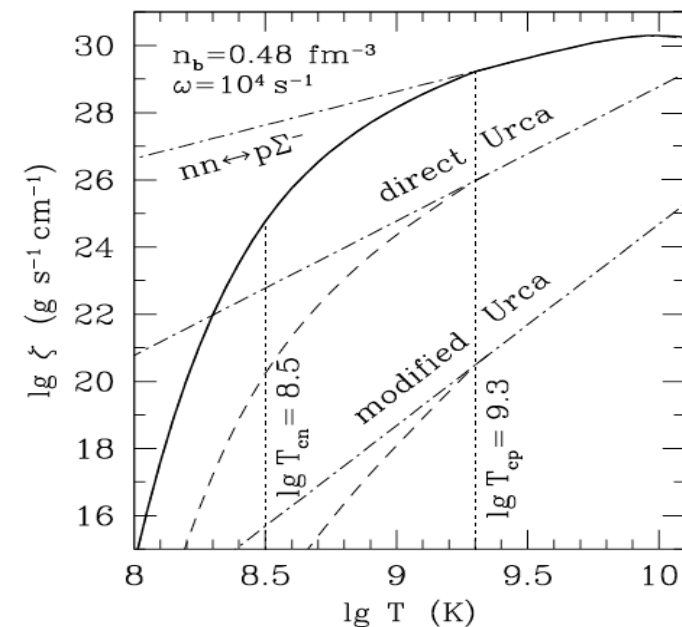


Fig. 3. Temperature dependence of bulk viscosity at $n_b = 0.48$ fm $^{-3}$ and $\omega = 10^4$ s $^{-1}$ in the presence of proton superfluidity with $\lg T_{cp} = 9.3$ and neutron superfluidity with $\lg T_{cn} = 8.5$. Dot-and-dashed lines (from up to down): partial bulk viscosities due to hyperonic, direct Urca and modified Urca processes, respectively, in non-superfluid matter. Associated solid and dashed lines: the same bulk viscosities in superfluid matter. Vertical dotted lines show $\lg T_{cn}$ and $\lg T_{cp}$.

Quark matter Alford, Schmitt, Rajagopal and Schaefer '08

there are different phases, such as CFL phase,
where all three colors and up, down and strange flavors
form conventional zero-momentum spinless Cooper pairs

CFL phase

-free-parameter predictions of QCD at
very large densities and rigorous consequences of
QCD in terms of few phenomenological parameters
at low densities

-all quark modes are gapped and relevant excitations are
Goldstone modes (phonons) for
thermal conductivity and viscosities
($\varphi \leftrightarrow \varphi\varphi$ $\varphi\varphi \leftrightarrow \varphi\varphi$ processes)

$$\kappa = \frac{2\pi^2 T^3}{45v^2} l_\varphi$$

$$l_\varphi \propto \mu^8 / T^9$$

$$\eta = 1.3 \times 10^{-4} \frac{\mu^8}{T^5}$$

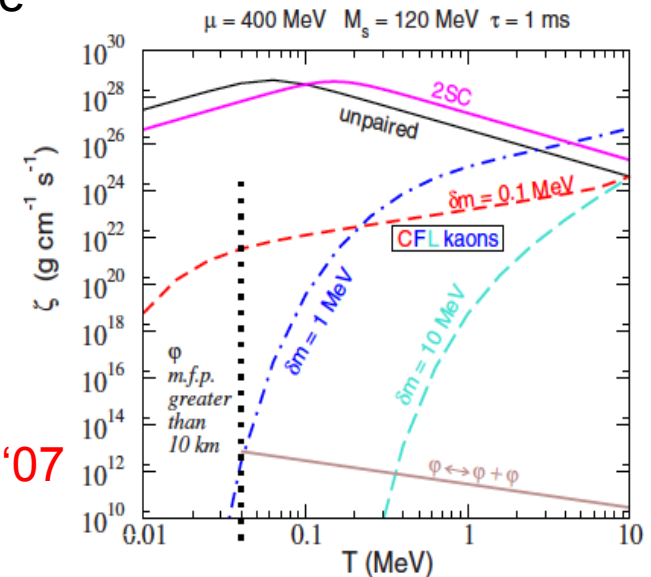
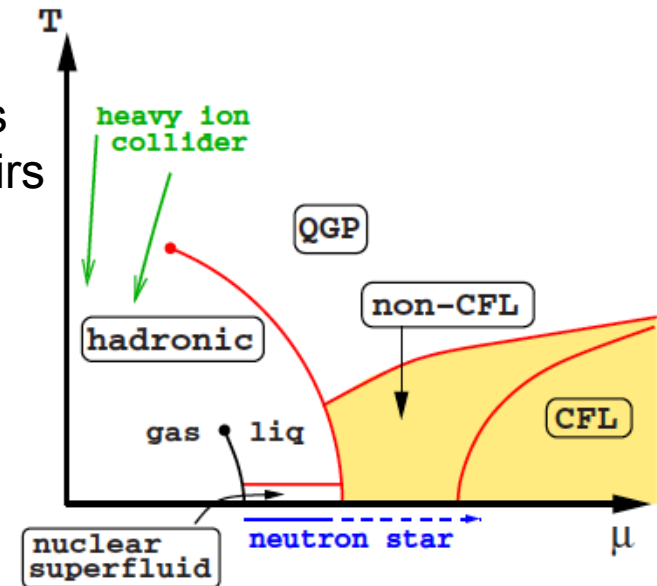
Manuel et al '05

Shovkovy & Ellis'02

$$\zeta = 0.011 \frac{M_s^4}{T}$$

Manuel &
Llanes-Estrada '07

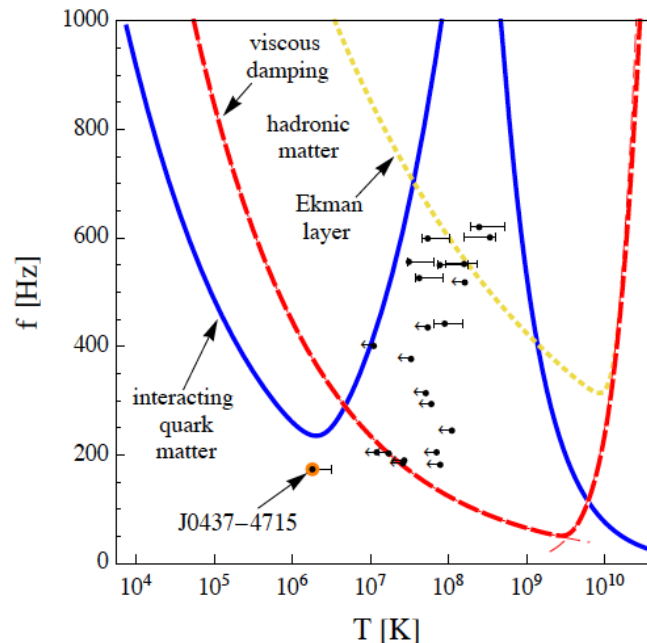
- electrical conductivity in CFL matter is dominated by
electrons and positrons (Shovkovy & Ellis '03)



comparison of bulk viscosity for CFL
with other phases

Possible observations to constrain transport properties of the core

- shear and bulk viscosities inside the neutron star core can help to understand r-mode instability and emission of gravitational waves



Data for accreting pulsars in binary systems (LMXBs) vs instability curves for **nuclear** and **hybrid** stars.

Alford & Schwenzer '13

- heat conduction problem in neutron star cores is needed to model **cooling of neutron stars** (especially in the first 100 years of their life), **thermal states of neutron stars in soft x-ray transients**, and **thermal relaxation of pulsars after glitches** (**taken from Shternin & Yakovlev '07 and references herein**)

Neutrino Transport

Yakovlev, Kaminker, Gnedin & Haensel '01

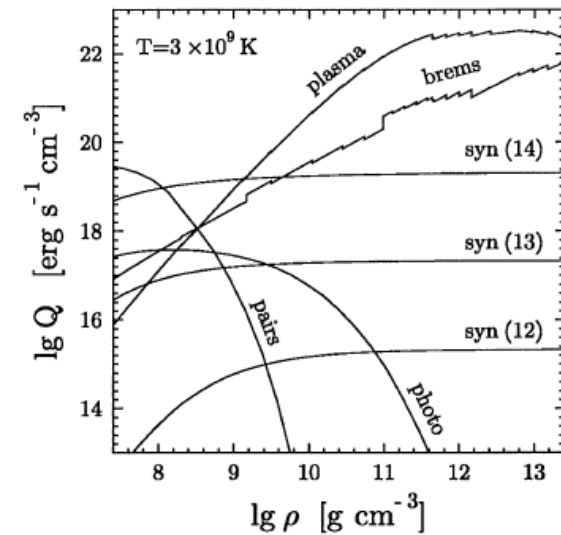
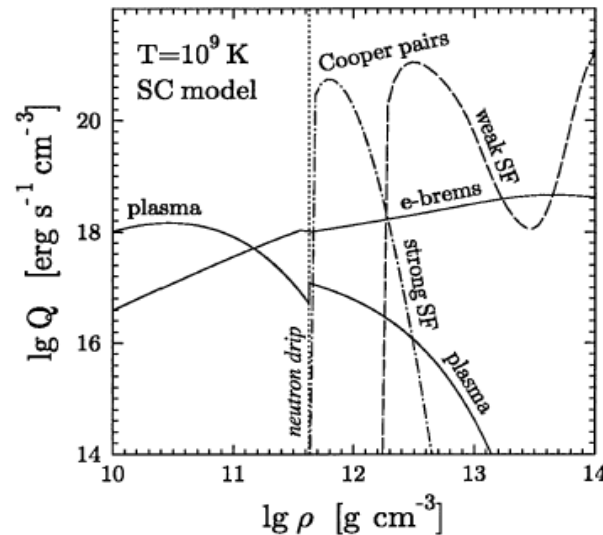
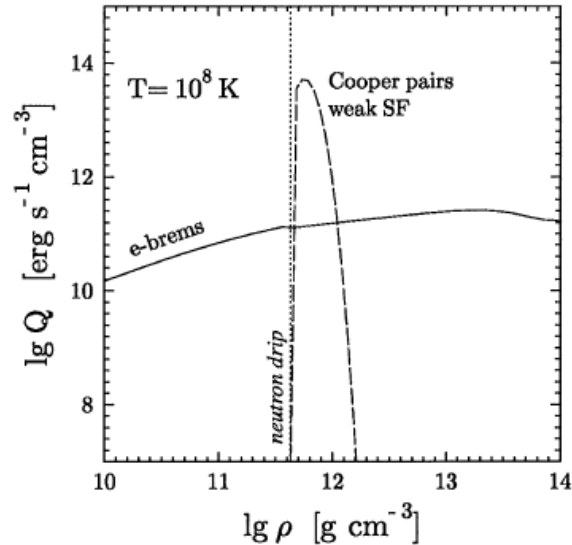
- important **sources of energy loss** in cooling of neutron stars **at initial stages** (10-100 yr)
- neutrinos interacting with very different states of matter (crust and core) under magnetic fields and/or superfluidity
- neutrino emission **for $\rho > 10^{10} \text{ gcm}^{-3}$ and $3 \times 10^6 \text{ K} \leq T \leq 10^{10} \text{ K}$**
- under these conditions:
 - electrons are a strongly degenerate ultrarelativistic ideal gas
 - atomic nuclei form strongly coupled Coulomb liquid or crystal, and may form a liquid crystal near the crust base
 - free neutrons in the inner crust are a strongly interacting Fermi liquid
- **neutrino emissivity Q** of the different processes based on Weinberg-Salam-Glashow theory of EW interactions

Crust

Main neutrino processes in a neutron star crust^a

No.	Process	
1	e^-e^+ pair annihilation	$ee^+ \rightarrow \nu\bar{\nu}$
2	Plasmon decay	$\gamma \rightarrow \nu\bar{\nu}$
3	Electron synchrotron	$e \rightarrow e\nu\bar{\nu}$
4	Photoneutrino emission	$e\gamma \rightarrow e\nu\bar{\nu}$
5	Electron-nucleus bremsstrahlung	$e(A, Z) \rightarrow e(A, Z)\nu\bar{\nu}$
6	Beta processes (including Urca)	$e(A, Z) \rightarrow (A, Z-1)\nu_e, (A, Z-1) \rightarrow (A, Z)e\bar{\nu}_e$
7	Cooper pairing of neutrons	$nn \rightarrow \nu\bar{\nu}$
8	Neutron-neutron bremsstrahlung	$nn \rightarrow nn\nu\bar{\nu}$
9	Neutron-nucleus bremsstrahlung	$n(A, Z) \rightarrow n(A, Z)\nu\bar{\nu}$

^a γ means a plasmon or photon; (A, Z) stands for an atomic nucleus.



Main contribution from 3 processes: plasmon decay at very high temperatures, and neutrino-pair bremsstrahlung and Cooper pairing of neutrons for $T \leq 10^9$ K

Core

a) in non-superfluid and non-magnetized neutron star cores

Main neutrino processes in $npe\mu\Lambda\Sigma^-$ matter^a

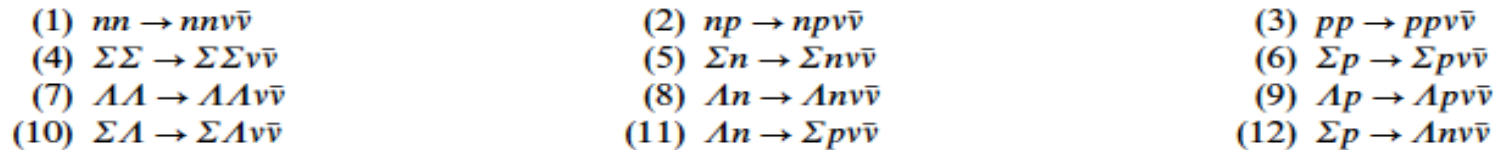
(I) Baryon direct Urca $Q \sim (10^{23}-10^{27})T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}$



(II) Baryon modified Urca $Q \sim (10^{18}-10^{21})T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$



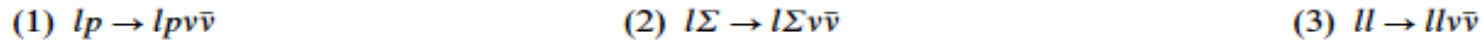
(III) Baryon bremsstrahlung $Q \sim (10^{16}-10^{20})T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$



(IV) Lepton modified Urca $Q \sim (10^{13}-10^{15})T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$



(V) Coulomb bremsstrahlung $Q \sim (10^{13}-10^{15})T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$



^a Σ means Σ^- ; l stands for e or μ ; B stands for n, p, Σ or Λ .

if allowed, baryon direct Urca processes are the strongest; otherwise, baryon modified Urca and baryon bremsstrahlung processes are the main reactions for cooling

Core

Leading neutrino processes in three models of exotic matter^a

Model	Process	Q (erg cm ⁻³ s ⁻¹)
Pion condensate	$\tilde{n} \rightarrow \tilde{p}l\bar{\nu}_l$ $\tilde{p}l \rightarrow \tilde{n}\nu_l$	$(10^{22}-10^{26})T_9^6$
Kaon condensate	$\tilde{n} \rightarrow \tilde{p}l\bar{\nu}_l$ $\tilde{p}l \rightarrow \tilde{n}\nu_l$	$(10^{22}-10^{24})T_9^6$
Quark matter	$d \rightarrow ue\bar{\nu}_e$ $ue \rightarrow d\nu_e$	$(10^{23}-10^{24})T_9^6$

^a l stands for e or μ ; \tilde{n} and \tilde{p} are quasinucleons (mixed n and p states); u and d are quarks.

direct Urca type in exotic matter with emissivities higher than mUrca in standard matter

b) in superfluid and magnetized cores

-**superfluidity**: a) reduces the emissivity of neutrino reactions and b) initiates specific neutrino emission due to Cooper pairing of baryons: **reduces the large difference between fast and slow cooling scenarios (and in certain conditions roles are interchanged)**

- **magnetic fields**: a) opening of neutrino reactions close at $B=0$ (**magnetic broadening**) and b) neutrino emission of charged particles moving in the magnetic field (such as neutrino synchrotron emission of electrons in uniform field)

Neutrino emission and neutron star cooling

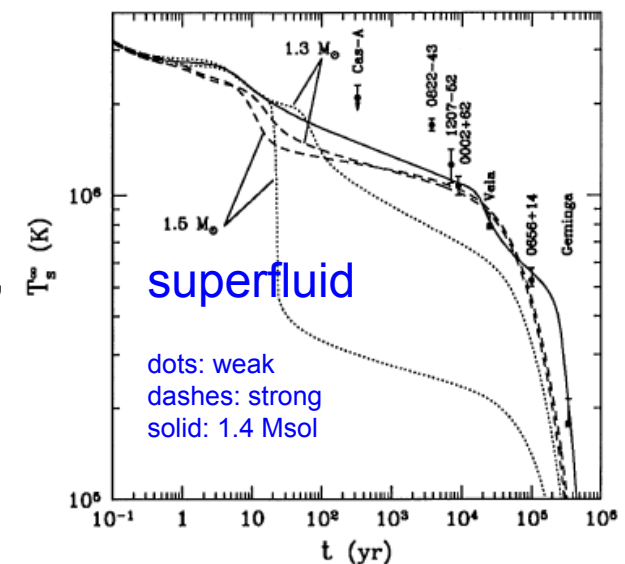
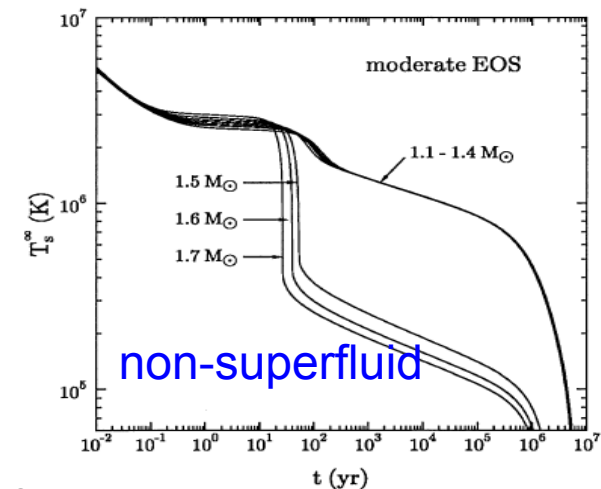
Cooling of non-superfluid neutron stars

1. $M < 1.44 M_{\odot}$: not high enough density to switch on dUrca. Standard cooling scenario with cooling curves independent of the stellar mass
2. $M > 1.44 M_{\odot}$: fast cooling scenario and spectacular drop of surface temperature for $t \sim 50$ yr

Cooling of superfluid neutron stars

1. superfluidity in the crust affects the cooling curves at the initial thermal relaxation stage, while superfluidity in the core affects cooling at later stages
2. superfluidity:
 - strongest **regulator of cooling**
 - may greatly **delay or accelerate the cooling**
 - **reduces** the large **difference between the fast and slow cooling** scenarios and, under certain conditions, it makes the slow cooling look like the fast and viceversa
 - **accelerates the cooling in the photon cooling era** by reducing the heat capacity of the stellar cores

Strong magnetic fields can enhance the cooling
(if stellar mass close to a certain threshold value)



DISCUSSION: Open questions

Transport phenomena in the CRUST

inner crust (Page & Reddy '12)

- existence and extent of **pasta phase** and transport properties (highly resistive layer to limit spin periods of X-ray pulsars by Pons, Viganò and Rea '13)
- **impurities** in the inner crust (need of nuclear structure studies)
- **nuclear excitations** relevant in the crust for $T \leq 10^{10}$ K?



Transport phenomena in the CORE

- **superfluidity/superconductivity** affecting transport coefficients
- neutron superfluidity can trigger **heat conduction via convective counterflow**
- **strong magnetic fields**: suppression of thermal conductivity due to Larmor rotation of charged particles as in electrical conductivity; affecting plasmon modes; magnetic fields with baryon superfluidity (Shternin & Yakovlev '07)

Neutrino Transport (Yakovlev, Kaminker, Gnedir & Haensel '12)

- mUrca process and baryon-baryon bremsstrahlung with **modern models of strong interactions**
- exact **superfluid reduction of mUrca** process and nucleon-nucleon bremsstrahlung **under superfluidity**
- **accurate analysis of neutrino emission** in reactions with nucleons **in the crust**