

# Gravitational wave emission from individual neutron stars

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## Gravitational wave emission from **individual neutron stars**

NSs in compact binary systems  
(see Giacomazzo, Hinderer, Bauswein, Galeazzi talks)  
do not belong to this class, **but**

- isolated NSs
  - NSs in Low Mass X-ray Binaries
- are included!**

Two mechanisms can lead to GW emission from individual NSs potentially detectable by 2<sup>nd</sup> and 3<sup>rd</sup> generation interferometers:

- Oscillations
- Deformations

In this talk I will discuss these two classes of processes, trying to answer to the key questions:

- Which processes can lead to **detectable GWs**?
- How can GW detection from these processes **constrain the EoS**?
- How can GW emission from these processes **explain astrophysical observations**?

## Neutron star oscillations

When a NS is excited by some perturbation, it can be set into *non-radial oscillations* emitting GWs at the characteristic frequencies of its quasi-normal modes (**QNMs**). These oscillations are *damped*, due to GW emission, thus have *complex frequencies*:

$$\omega_n = \sigma_n + i / \tau_n$$

The quasi-normal modes **frequencies** ( $\sigma_n$ ) and **damping times** ( $\tau_n$ ), at which the NS oscillates, are also the frequencies and damping times at which it emits **gravitational waves**.

Several possible excitation mechanisms:

- glitches
- gravitational collapse giving birth to the NS
- accretion from a companion star
- electromagnetic activity, as in magnetar giant flares
- phase transition of the matter composing the star
- compact binary inspiral and coalescence (see other talks)

# Neutron star oscillations

Few further remarks:

- **GWs are the perfect tool to study QNMs of NSs:**
  - GWs require strong-field phenomena, such as those in the inner core of a NS, to be generated;
  - once produced, GWs hardly interact with matter and energy, carrying unaltered information on the emitting process.
- **NS QNMs carry the imprint of the behaviour of ultra-dense matter**  
The measurement of frequencies and damping times of the QNMs would give us unvaluable information on the matter composing the star, in particular on the EoS of matter at supra-nuclear densities (*gravitational wave asteroseismology*) but also other features (superfluidity, thermodynamical gradients, etc.)
- **Some QNMs can become unstable**  
in presence of rotation, dynamical and secular instabilities can set in; could lead to a large GW flux, and could explain observed values of NS spins. Oscillation instabilities also appear when the NS is warm, soon after birth.

# Neutron star oscillations

The QNMs of NSs are classified depending to the *main restoring force*:

- **g-modes**: buoyancy force. Probe entropy/composition gradients.  
 $\nu \sim$  few hundreds of Hz but  $\tau$  very large  $\Rightarrow$  not efficient in radiating GW
- **p-modes**: pressure.  $\nu \sim$  few thousands of Hz. Probe sound speed in the star.
- **f-mode**: the fundamental mode, intermediate between g and p.  $\nu \sim 1$ -2 kHz; it the most efficient in radiating GW ( $\tau \sim 1$  s). Scales  $\sim$  with average density.
- **w-modes**: pure space-time modes.  $\nu \sim$  many kHz

Including more physics, other families of modes appear:

- **r-modes**: Coriolis force (for rotating stars only)
- **magnetic modes**
- **elastic modes** (in the crust)
- **superfluid modes** (when baryon superfluidity occurs)

Several groups (e.g. Tübingen, Southampton, Rome, Thessaloniki, St. Petersburg, etc.) are working to include more and more physics, to study different kinds of QNMs

# Neutron star oscillations

I will first summarize the state of the art about the study of **stable** modes, in the context of GW asteroseismology.

It should be remarked that with this approach we can compute **frequencies** and **damping times** (given the EoS and other NS properties) but we know very few about the **amplitude** of the modes, i.e. their excitation.

What do we know about excitation mechanisms and amplitudes for stable modes:

- QNM of a proto-NS excited by **core collapse**: collapse modelling still uncertain, SN signal may be dominated by PNS oscillations ( $h \sim 10^{-23} - 10^{-20}$  for galactic SN)  
*(Ferrari '10; Scheidegger et al. '10; Ott et al. '11)*
- QNMs from NS **glitches** (sudden changes in NS rotation rate, driven by interaction of internal superfluid with the NS crust, or crust reconfiguration). Current models *(e.g. Sidery, Passamonti, Andersson '10; Haskell, Pizocchero, Sidery '12)* predict amplitudes too low for detection by 2<sup>nd</sup> and 3<sup>rd</sup> generation interferometers.
- QNMs in **binary** NSs (tidal excitation and hypermassive NS), see other talks

**Unstable** modes will be discussed later

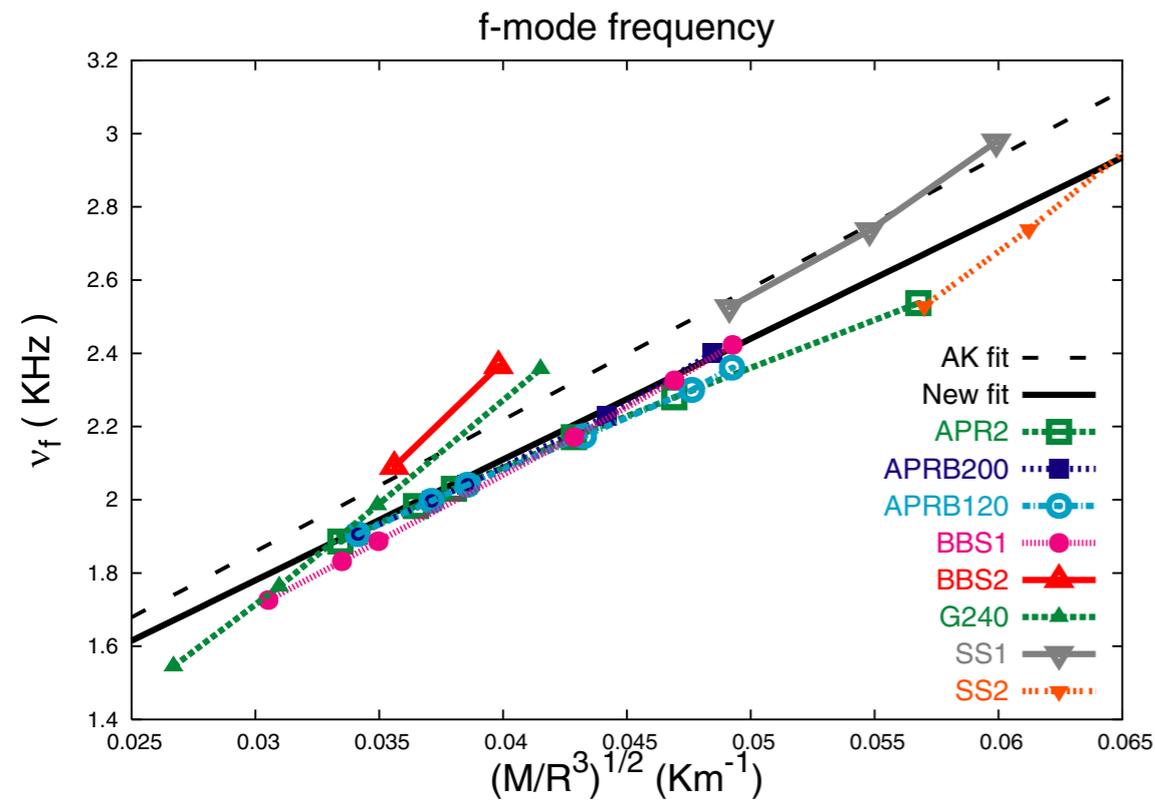
# Neutron star oscillations

## GW asteroseismology of simple (cold, non-rotating, non-superfluid etc.) NSs

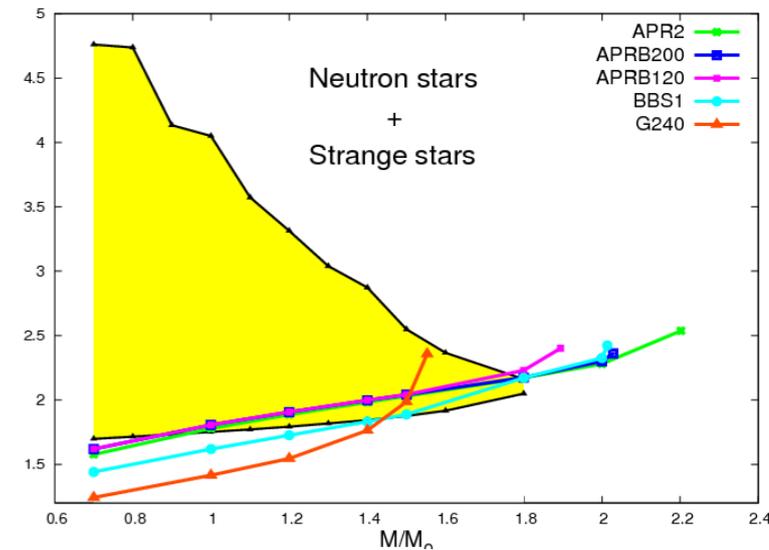
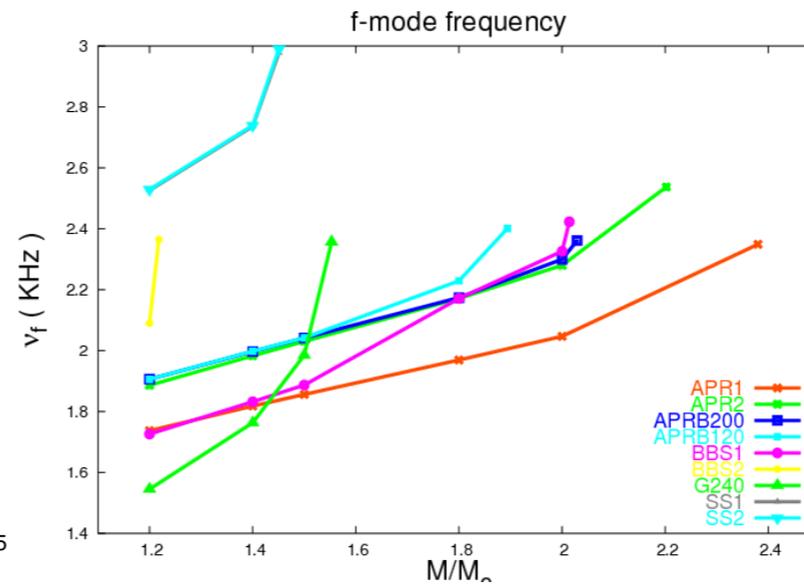
(N.Andersson & K.Kokkotas, MNRAS '98; K.Kokkotas, T.Apostolatos, N.Andersson, MNRAS '01; O. Benhar, V. Ferrari, L.G., S. Marassi '04, '07)

QNMs of NSs are functions of their mass and radius, irrespective of the EoS.

If we measure (through GW detection) the frequencies and damping times of the f- and p<sub>1</sub>- modes, we know M and R, useful to understand the NS EoS



(O Benhar et al., '04, '07)

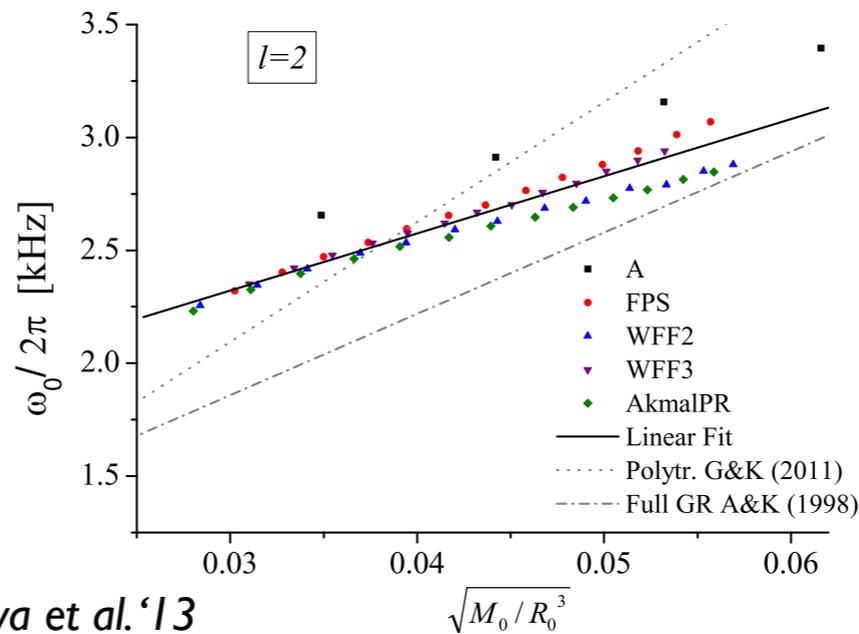
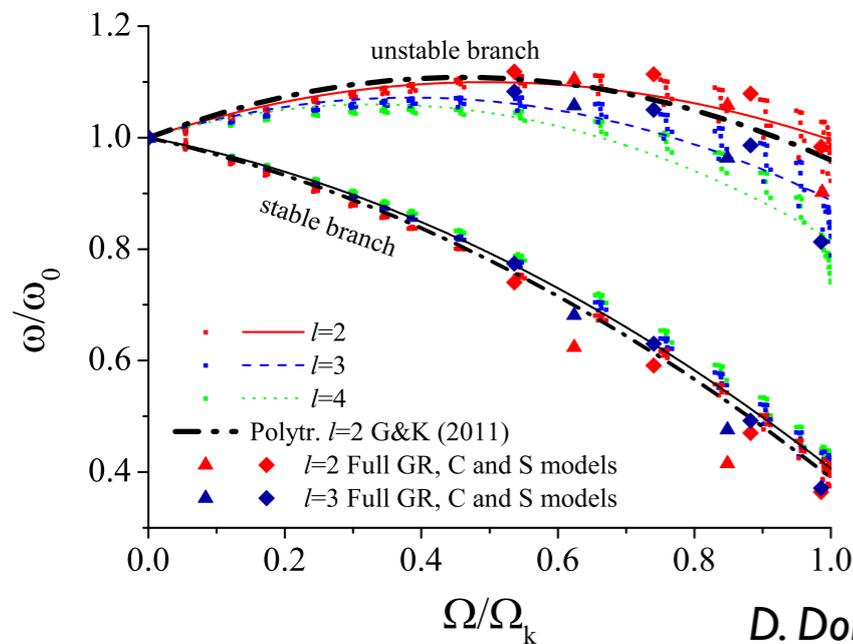


Alternatively, a simple measure of the f-mode frequency (if M is known) can significantly constrain the EoS

# Neutron star oscillations

## ● GW asteroseismology of **rotating NSs** (stable modes)

(H. Dimmelmeier, N. Stergioulas, J. Font '06; W. Kastaun, '08; E. Gaertig, K. Kokkotas '11; S. Yoshida '12; D. Doneva, E. Gaertig, K. Kokkotas, C. Kruger '13)



Fits with  $M, R$  still hold but in two steps:

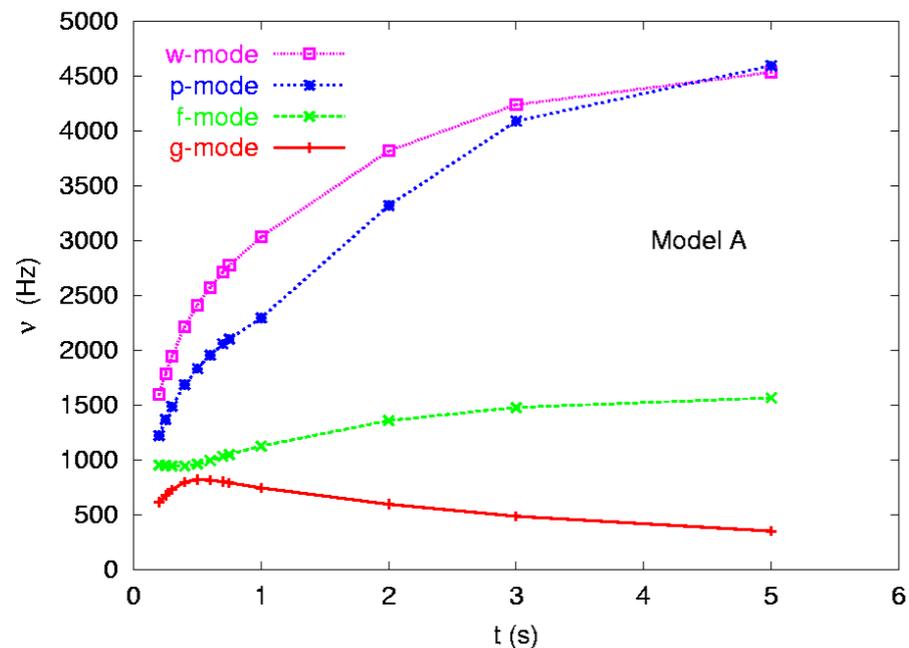
first find (given  $\Omega$ ) the corresponding QNM frequency of the non-rotating star,  
then, from this, get  $M/R^3$ .

Still, only approximated solutions (Cowling or CFC or  $m=0$ , etc.)

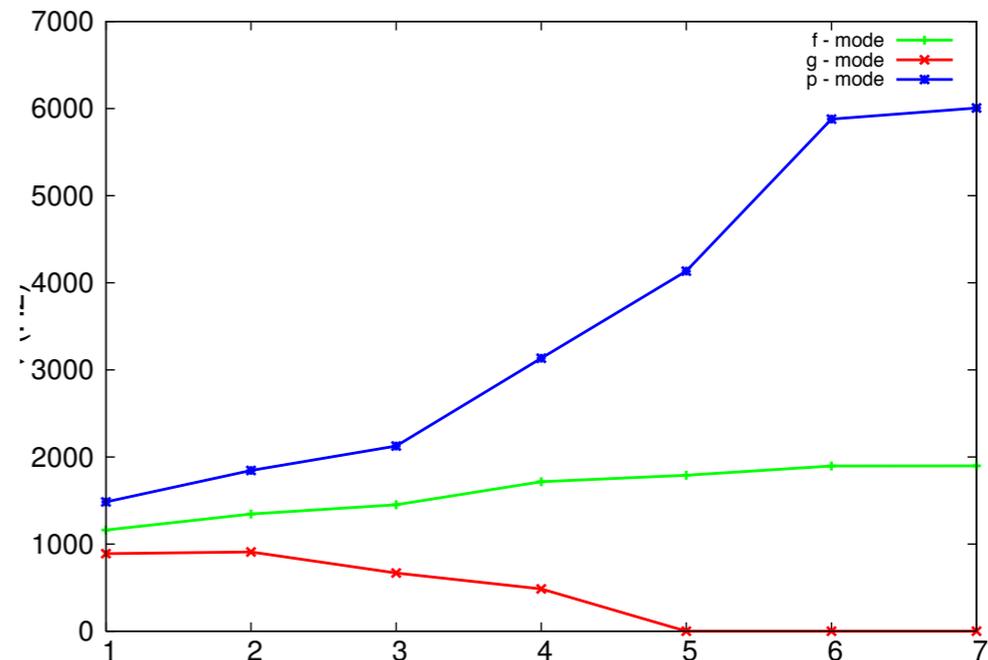
# Neutron star oscillations

## GW asteroseismology of hot, newly born proto-neutron stars

In the first minute after the supernova explosion, a fraction of the kinetic energy excites QNM oscillations, whose frequencies and damping times are affected by the thermodynamical and neutrino properties of the star.



(V.Ferrari, J.A. Pons, G. Miniutti, MNRAS '03)



(F. Burgio, V. Ferrari, L.G., H.-J. Schultze, PRD '11)

While in cold, old NSs the f- and p- modes are the only ones relevant for GW emission, in hot, young NSs the g-mode, which is related to the entropy gradient, is important as well.  $\sigma$  and  $\tau$  of the g- and f- mode tend to converge in the first tenths of s.

# Neutron star oscillations

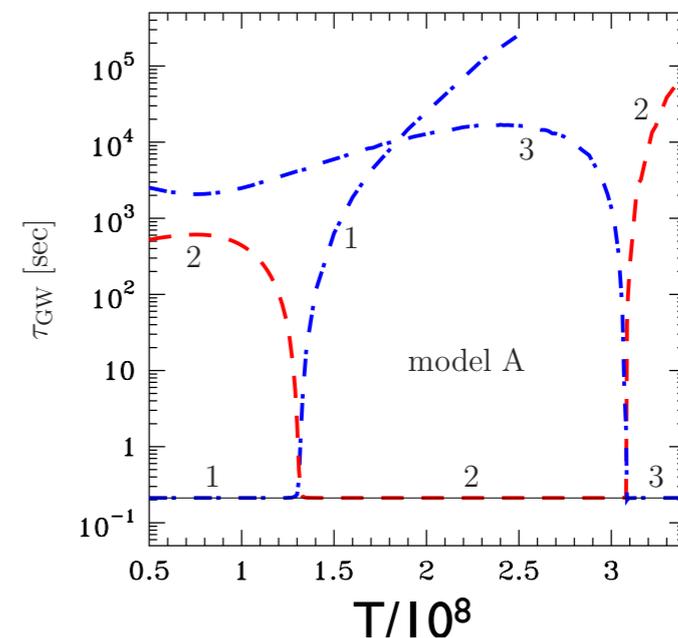
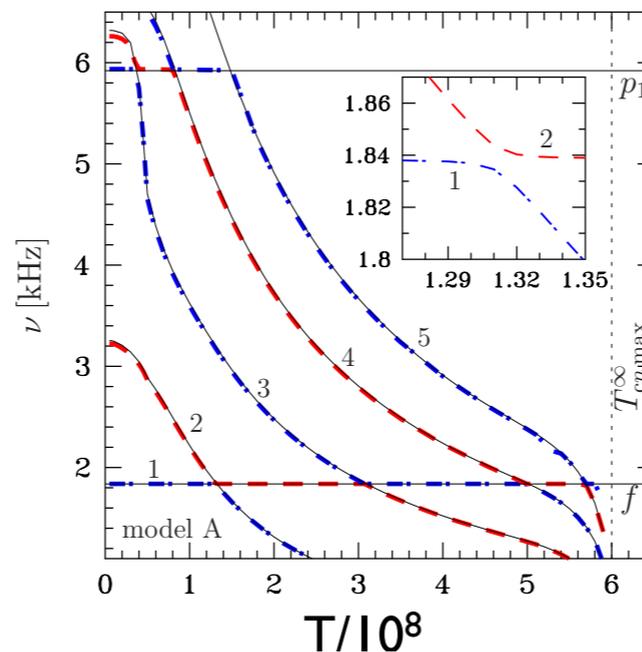
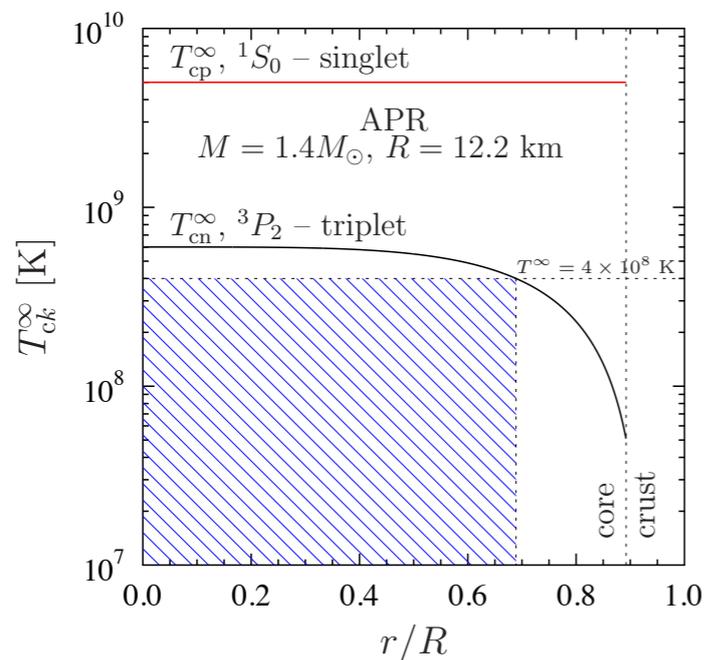
- GW asteroseismology of NSs with **baryon superfluidity in the core**

(S.Yoshida, U.Lee, PRD '03; L.Lin, N.Andersson, G.Comer, PRD '08; A.Chugonov, M.Gusakov, MNRAS '11; L.G., E.Kantor, M.Gusakov, A.Chugunov, PRD '14)

In recent years, theoretical computations and observations are providing a growing evidence that baryons in the NS core are **superfluid**.

To include superfluidity of neutrons and superconductivity of protons in the model of NS oscillations, the star should be treated as composed by two (interacting) fluids.

A new class of QNMs appears, with a strong dependence on temperature; at some  $T_i$  they become similar to the other modes: GW emission can be relevant.



L.G., E. Kantor, M. Gusakov, A. Chugunov, PRD '14

# Neutron star oscillations

- GW asteroseismology of NSs with **magnetic fields and elasticity in the crust**

(L. Samuelsson, N. Andersson, MNRAS '07; Y. Levin, MNRAS '07; H. Sotani, K. Kokkotas, N. Stergioulas, MNRAS '08;

P. Cerda-Duran, N. Stergioulas, J. Font, MNRAS '09; A. Colaiuda, K. Kokkotas, MNRAS '11; M. Gabler, P. Cerda-Duran, J. Font, E. Muller, N. Stergioulas, MNRAS '13)

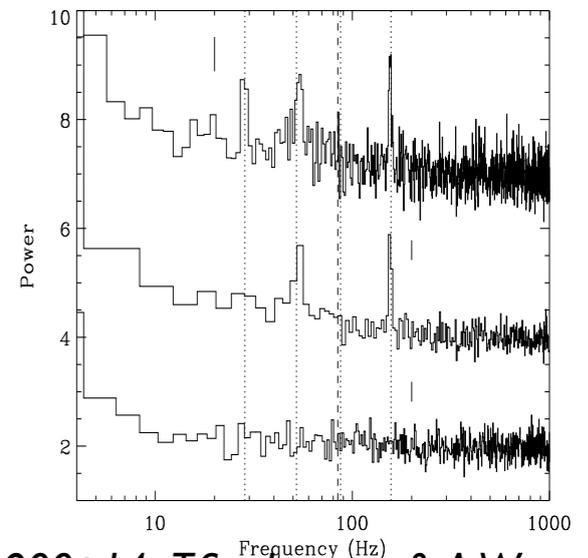
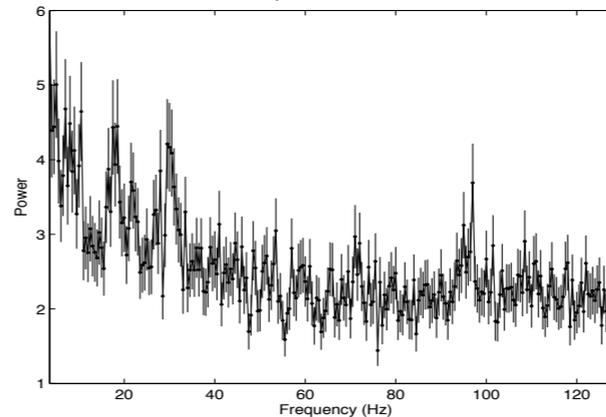
Real stars do have **magnetic fields** (up to  $\sim 10^{12}$  G for normal stars,  $\sim 10^{16}$  G for magnetars) and **elasticity in the crust** (shear modulus  $\mu \sim 10^{30}$  erg/cm<sup>3</sup>).

Each of these ingredients yields a new family of modes (Alfven and torsional), with frequencies from few tens to few hundreds Hz.

The spectrum of magnetic modes seems to be quite involved: there are both a continuum band, and discrete modes.

They can probably explain QPOs in the X-ray emission from giant flares in magnetars (presently, the only observational data of NS QNMs!)

SGR 1806-20, G. Israel et al. '05



SGR 1900+14, T. Strohmayer & A. Watts '05

These oscillations seem to be due to coupled magnetic and torsional modes of the crust, possibly coupled with the core.

Not clear whether they can emit detectable GWs

(Levin '06; Corsi & Owen '11; Y. Levin & M. van Hoven '11; D. Zink, P. Lasky, K. Kokkotas '12)

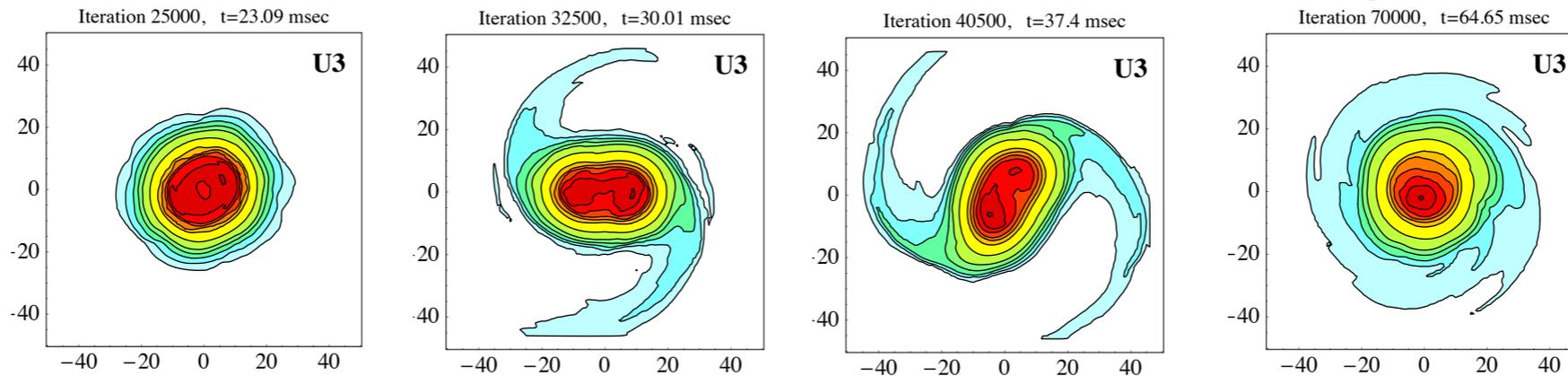
# Instabilities of rotating neutron stars

Rotating neutron stars can have **instabilities** appearing, at the linear level, as **unstable QNMs**.

$$\text{Effective if } \frac{1}{\tau_{inst}} < \sum_i \frac{1}{\tau_{diss}^i}$$

- Dynamical instabilities: growth time of the order of the oscillation timescale

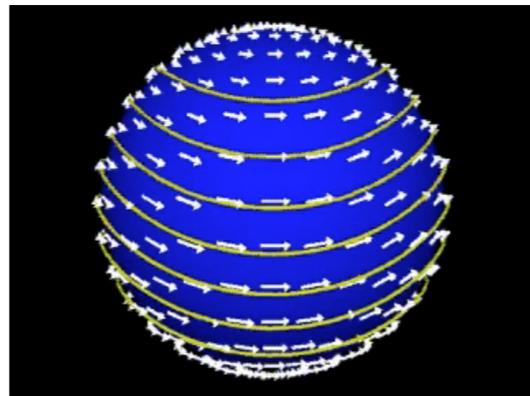
Most relevant for GW emission: **bar-mode instability** of f-mode



(Baiotti et al. '07)

- Secular instabilities: growth time much larger than the oscillation timescale

Most relevant for GW emission: **CFS instability** of r-modes



(Univ. Illinois Urbana-Champaign)



# Instabilities of rotating neutron stars

## Secular instabilities

(Bondarescu et al., '07; Watts et al., '08; Haskell et al. '09; Passamonti et al. '09; Ho et al., '11; Haskell et al. '12; Doneva et al. '13; Makmoodifar & Strohmayer '13; Bondarescu & Wasserman '13; Alford et al. '14; Gusakov et al. '14; Haskell et al. '14)

CFS instabilities are GW-driven secular instabilities which set in when the propagating mode of a rotating star has a pattern speed which is co-rotating with respect to infinity, counter-rotating with respect to the star.

In this case, GW emission removes angular momentum, but the amplitude of the perturbation **increases** (at the expense of the NS rotational energy).

The amplitude of the mode keeps increasing, until **dissipative effects** set in, or, due to non-linear couplings, the mode reaches a **saturation amplitude**.

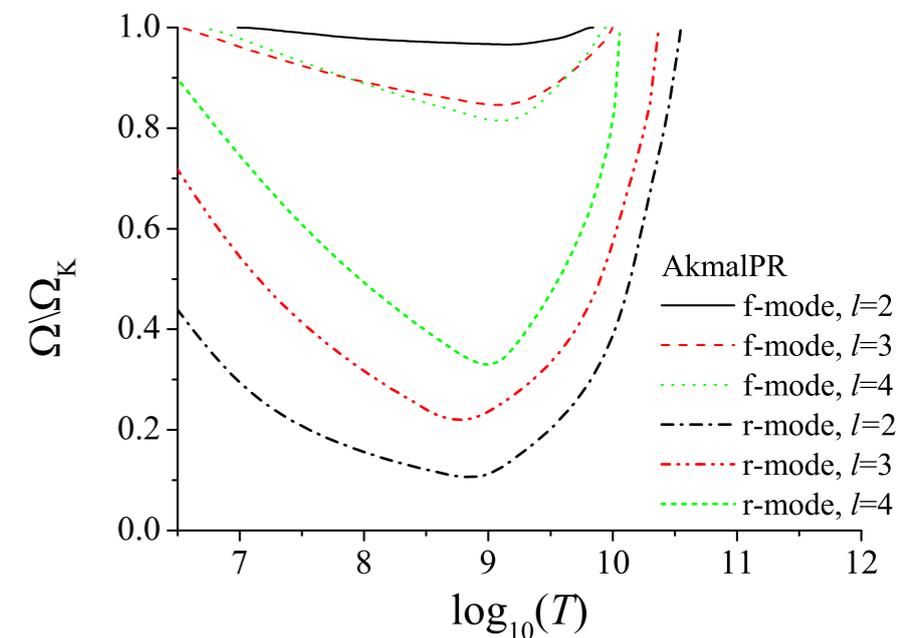
CFS instability of **f-mode** requires rapid rotation:

may be relevant in core-collapse;

as in the case of dynamical instability, it may or may not

be associated to detectable GWs: we need to understand better the features of core-collapse.

CFS instability of **r-modes** is more interesting: it occurs at **any** value of  $\nu$ , and could also occur in accreting NSs in low-mass X-ray binaries (LMXBs).



(Doneva et al. '13)

# Instabilities of rotating neutron stars

**r-mode instability in LMXBs** (see Ho's talk)

This instability may explain why observed NSs in LMXB (and MSRP) have  $\nu \lesssim 720\text{Hz}$ , significantly smaller than the maximum (breakup) rotation rate.

If r-mode instability is responsible to this bound, it could be associated to significant GW emission from NSs in LMXBs (but even in this optimistic case, GW searches may be difficult as long as we do not know M and  $\nu$  of NSs in LMXB).

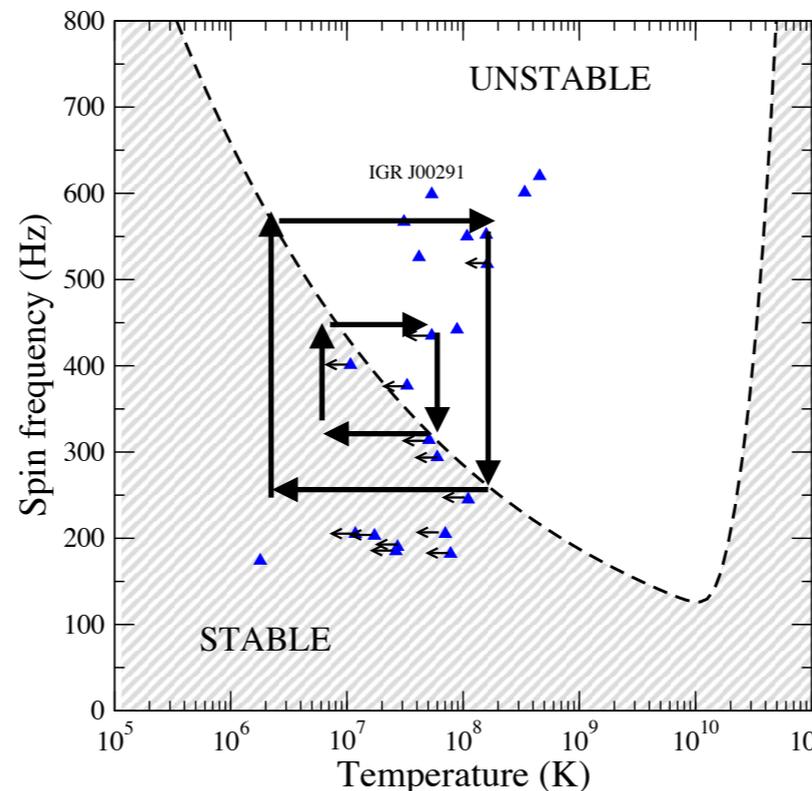
However, there is a problem:

r-mode instability window

$$\left( \frac{1}{\tau_{inst}} = \sum_i \frac{1}{\tau_{diss}^i} \right)$$

and T,  $\nu$  of observed NSs

smaller  $\alpha$



(Haskell et al. '14)

possible cycle of r-mode development ( $\alpha \gtrsim 10^{-3}$ ):

- instability heats
- GW emission spins down
- in stable region, cools
- accretion spins up

NS spends  $\approx 1\%$  of time in unstable region!

# Instabilities of rotating neutron stars

## r-mode instability in LMXBs

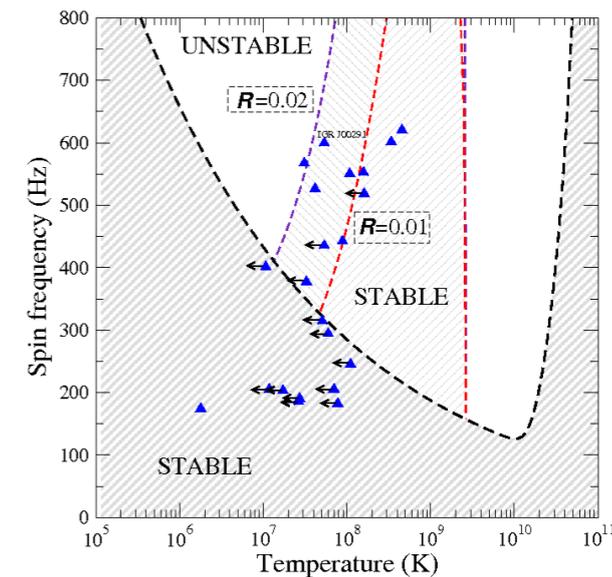
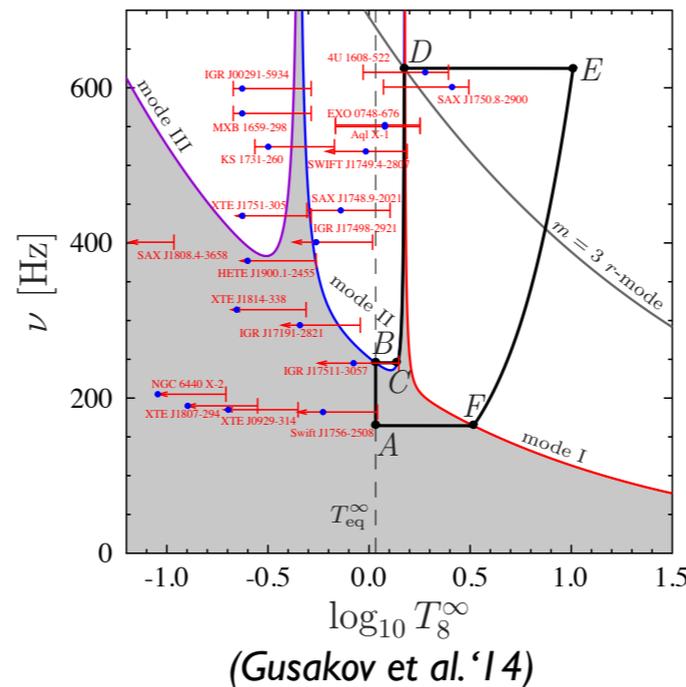
Possible ways to reconcile theory with observations:

- Saturation amplitude  $\alpha$  is very small, r-mode instability does not affect the T-v evolution. Spin limit would be due to other mechanisms (e.g., disk-magnetosphere interaction). In this case, the GW emission would be weak.

- Additional physical ingredients provide further damping that changes the instability window; for instance:

- superfluid mutual friction

- coupling of r-modes with superfluid modes



In both cases  
**large GW emission**  
 is possible!

# Neutron star deformations

Rotating NSs emit GWs if they are not symmetric with respect to symmetry axis with frequency  $\nu_{\text{rot}}$  and  $2\nu_{\text{rot}}$  (potentially in detectors bandwidth).

Deformations can be:

- triaxiality
- misalignment between rotation and symmetry axes (“wobble angle”)
- localised deformations (“mountains”)

All of them can be described by **quadrupole ellipticity**

$$\epsilon_Q = \frac{Q}{I} \quad \leftarrow \begin{array}{l} \text{mass-energy quadrupole deformation} \\ \text{momentum of inertia} \end{array}$$

the emission is of the order  $h \sim \frac{16\pi^2 G \nu^2 I}{c^4 r} \epsilon_Q$  **this emission could explain spin limit in LMXB!**

Deformation can be due to:

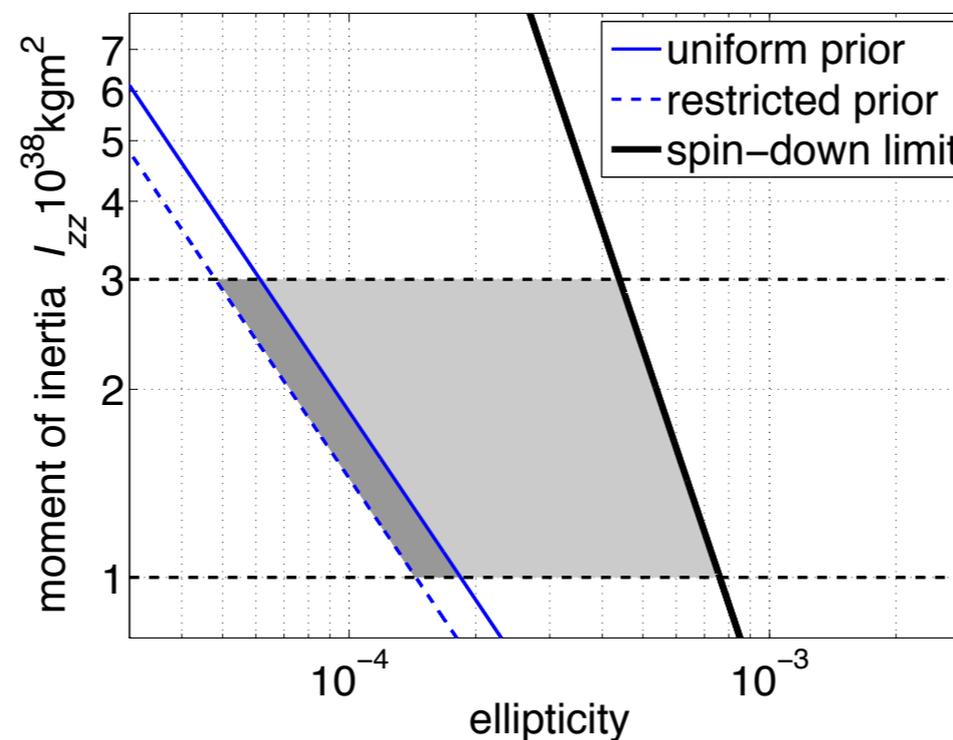
- accretion (heating the crust and perturbing the magnetic field)
- primordial magnetic field

# Neutron star deformations

## LIGO/Virgo bounds

In some cases, best bounds on NS deformation come from 1<sup>st</sup> generation GW detectors

Crab pulsar  
(similar results have been obtained for Vela pulsar)



(Abbot et al. '08)

No more than 2%  
of the spin-down energy  
is emitted through GWs!

Previous bound came from spin-down limit:  $\epsilon_Q \leq \left( \frac{5c^5 |\dot{\nu}|}{512\pi^4 G\nu^5 I} \right)^{1/2}$

Bounds from GW non-detection by LIGO/Virgo are stronger!

# Neutron star deformations

## Maximal deformation

(B.Haskell, D.Jones, N.Andersson, MNRAS '06; B.Owen, PRL '05; C.Horowitz & K.Kadau, PRL '09;)

There is a maximal deformation that the crust can sustain before breaking.

Standard scenario suggests  $\varepsilon_Q \approx 10^{-5} - 10^{-6}$

but some “exotic matter” allows for larger values

(note that  $\varepsilon_Q \sim 10^{-6}$  would mean a “mountain” as high as  $\sim 2\text{cm}$ !),

e.g.  $\varepsilon_Q \approx 10^{-4}$  for a strange quark star,  $\varepsilon_Q \approx 10^{-3}$  for a color superconducting quark star.

Alternatively, if a newly born proto-NS reaches stationary configuration with deformations (due to magnetic fields) *before* crust formation, the deformation could persist (*but how long?*) overcoming the above bounds.

# Neutron star deformations

## Accreting mountains

(B.Haskell, D.Jones, N.Andersson, MNRAS '06; M.Prymak, A.Melatos, D.Payne, MNRAS '11; N.Johnson-McDaniel & B.Owen, PRD '13)

- Thermal mountains

Due to crust heating as the accreted material sinks into the crust, increasing density and starting nuclear reactions.

Difficult to estimate the magnitude of the deformation (many parameters in the model).

- Magnetic mountains

Due to local perturbations of the magnetic field structure by accreting material, leading (depending on the EoS) to

$$10^{-8} \lesssim \epsilon_Q \lesssim 10^{-4}$$

(note that the larger value could crack the crust, and can be competitive with LIGO/Virgo bound)

# Neutron star deformations

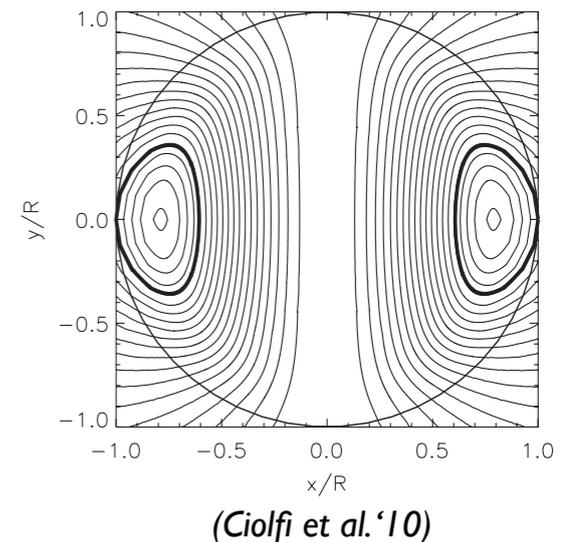
## Deformations from primordial magnetic field

(B.Haskell, L.Samuelsson, K.Glampedakis, N.Andersson, MNRAS '08, A. Colaiuda, V.Ferrari, L.G., J.Pons, MNRAS '08;  
S.Lander & D.Jones, MNRAS '09; R.Cioffi, V.Ferrari, L.G., MNRAS '10;)

Magnetic field in a newly-born proto-NS can deform the star:

$$\epsilon_Q \simeq k 10^{-4} \left( \frac{B_{pole}}{10^{16} G} \right)^2 \quad (k \sim 4-9 \text{ depending on the EoS})$$

B is expected to have a mixed toroidal-poloidal character (*twisted torus*)



- this mechanism requires wobble angle, which decays as  $\tau_d \sim \frac{nP}{\epsilon_Q}$  ( $n \sim 10^2-10^5$ )  
=> up to few years for large B
- large B would spin down the NS to  $\nu$  outside detector bandwidth in a few years  
=> too rare for detection (these GWs are detectable for galactic sources)  
**but** accretion from a companion could keep large spin and wobble for a longer time, raising the rate and making this signal detectable.

# Neutron star deformations

Previous estimates were based on simplified modelling of magnetic field, in which it was treated “classically”, but in presence of a superfluid the picture is different.

( *K.Glampedakis,D.Jones,L.Samuelsson, PRL '12; K. Henriksson & I.Wasserman, MNRAS '13; S.Lander PRL '13, MNRAS '14* )

If type II superconducting protons, deformation linear in B

$$\epsilon_Q \sim 10^{-4} \left( \frac{B}{10^{16} \text{ G}} \right) \left( \frac{H_{crit}}{10^{16} \text{ G}} \right)$$

allows for smaller values of magnetic field.

For color-magnetic stars, deformation is much larger: even the signal from Crab and Vela may be detectable by ET.

# Conclusions

Oscillating and deformed NSs are potential sources of GWs for 2<sup>nd</sup> and 3<sup>rd</sup> generation detectors, but there is still work to do to model emitting sources.

- Stellar oscillations contain precious information on the NS EoS.
  - Different classes of QNMs correspond to different physical features, which should be included in the model.
  - It is difficult to estimate the amplitude of stable modes.
  - Unstable modes are a promising source of GWs, for newly born NSs and for accreting NSs in LMXB.
  - Unstable modes could also explain the spin limit of NSs.
- Stellar deformations, due to accretion or to primordial magnetic fields, are also a potential source of GWs, but we do not know yet the amplitude of the deformation.