COST Action MP1304 STSM Report

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Purpose

The goal of the STSM was to develop simplified phenomenological lattice models for neutron stars based upon the assumption of the superfluidity of the neutrons in the crust. The microscopic motivation of the considered lattice models is provided by the effective neutron-neutron interactions and the interaction of the neutrons with the protons in the crust. The main goal of my visit, performed with Andrea Trombettoni, was the determination of the parameters of the effective models from microscopic results.

Description of the work and main results

During my stay it was discussed on how one can model the inhomogeneity present in the neutron star crust to build an effective XY model. A discussion of possible lattice models and their motivations is presented in the Report of Andrea Trombettoni. The inhomogeneity is introduced through the couplings J_{ij} in the Hamiltonian

$$H = -\sum_{\langle i,j\rangle} J_{ij} \cos\left(\varphi_i - \varphi_j\right). \tag{1}$$

In (1) $J_{ij} = \sqrt{n_i n_j} t_{ij}$ where n_i is the number of neutrons in the cell i and t_{ij} is the hopping parameter between the nearest neighbour sites i and j. The XY model (1) is affected both from the fact that the size of the Wigner-Seitz cell is increasing as we go deeper in the crust (closer to the center), and also the potential itself inside the Wigner-Seitz cell depends on the radial coordinate. It was discussed how to determine the parameters needed to compute the inhomogeneity and how to use them in the actual computation.

The numbers n_i can be computed using the results contained in several papers done by the Coimbra group, including S. S. Avancini, D. P. Menezes, M. D. Alloy, J. R. Marinelli, M. M. W. Moraes, and C. Providencia, *Warm and cold pasta phase in relativistic mean field theory*, Phys. Rev. C **78**, 015802 (2008); F. Grill, C. Providencia, and S. S. Avancini, *Neutron star inner crust and symmetry energy*, Phys. Rev. C **85**, 055808 (2012); and F. Grill, H. Pais, C. Providencia, I. Vidana, and S. S. Avancini, *Equation of state and thickness of the inner crust of neutron stars*, Phys. Rev. C **90**, 045803 (2014) [and references therein].

The results are shown in Figs.1-2. In Fig.1 we plot n_i for the considered 30 equispaced cells in the inner crust (the last two are omitted since the density is too small). To compute the tunneling rates one needs to compute the one-body potential V_{1b} introduced and discussed in the companion Report of Andrea Trombettoni. In that Report the potential V_{1b} is also plotted. A first estimate of t_{ij} can be obtained using V_{1b} and an estimate for the Wannier wavefunctions. For a given external potential we can fix the Wannier function by a variational ansatz which should minimize the functional energy, containing the potential term given by V_{1b}. In the following L is the periodicity of the lattice and a is the width of the well, estimated from V_{1b}. Assuming a Gaussian form for the Wannier function, and again within the tight-binding approximation, we can fix the parameters of the Gaussian and therefore we have all the ingredients necessary to compute t_{ij} for each Wigner-Seitz cell.

Using a periodic square well potential as an approximation we calculated the corresponding t_{ij} to hop between two consecutive wells. The potential has three parameters: the width of the well a, the periodicity of the potential L and the depth of the potential V. In Fig.2 we plot the value of t_{ij} in units of the reference energy $E_r = \frac{\hbar^2}{2ma^2}$ for some depths and as a function of the dimensionless parameter L/a. Actually, for a cell in the center of the crust (~ 15) it is $V_o/E_R \approx 250$.

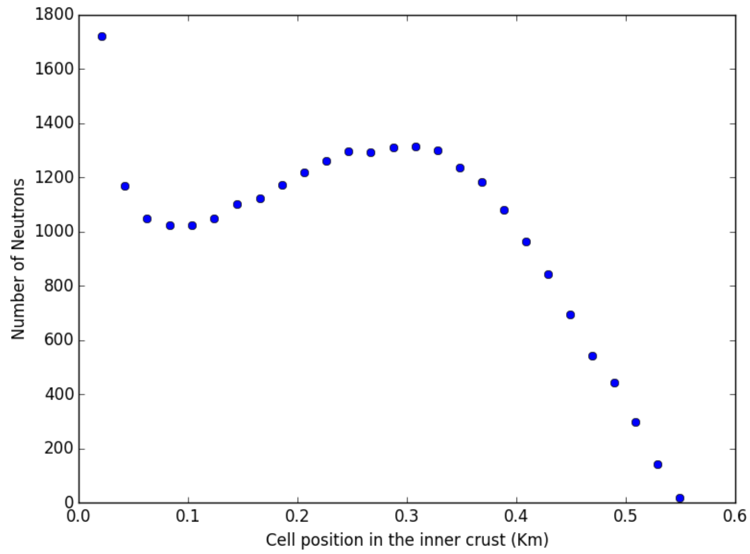

Figure 1: Number of neutrons n_i as a function of the distance from the beginning of the inner crust (i.e, closer to the center).

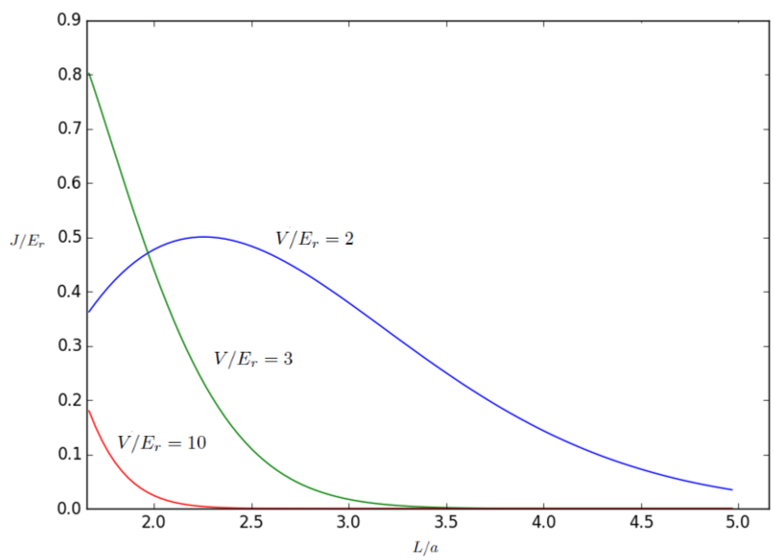

Figure 2: Tunneling rate using a radial periodic well potential. L is the periodicity of the lattice, a is the width of the well. The energy is measured in units of the reference energy $E_r = \frac{\hbar^2}{2ma^2}$.

With the code written in Coimbra, the t_{ij} may be obtained using the numerical data for V_{1b} provided by the Coimbra group. These results have been checked with the hoppings computed using the actual numerical data for V_{1b}. The variation of lattice spacing is known from V_{1b}. The lattice spacing in the phenomenological model is two times the Wigner-Seitz cell radius and it is dependent on the density. The way the density profiles depend on the neutron star mass is shown in the papers previously cited. The data used to calculate the hopping coefficient come from the FSUGold parametrization on the Walecka model for nuclear interactions with the mass equal $1.4M_{\odot}$. It was decided to build a system of $30 \times 30 \times 30$ cells, so the heart of the above calculation resides in calculation of 29 hopping coefficients along the radial direction. We observe that a more accurate estimate of t_{ij} can be obtained using the effective two-body neutron-neutron potential, which can be included in the functional energy to be minimized.

We also had some preliminary discussions on building a purely fermionic lattice model and on how to study it in equilibrium through mean-field approach. This model is less phenomenological since it does not assume, since from the beginning, the superfluidity of neutrons present in the first model, however it appears not easily treatable.

Future

In the future we expect to continue this collaboration making use of the different backgrounds and expertise to study properties of the neutron stars. In an immediate future we foresee to further improve the estimates for the hoppings (which, at this level, are single-particle) and to study in detail the above inhomogeneous XY model. After I plan to extend the joint work to the second purely fermionic model discussed above. Personally I am always eager to learn and work on interdisciplinary projects and therefore this constitutes an extra personal motivation to maintain this collaboration in the future.