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STSM RESEARCH PROJECT REPORT

A new relativistic approach for nuclear equation of state and application to neutron star physics
COST-STSM-ECOST-STSM-MP1304-261015-063615
26/10/2015 – 4/12/2015
Host: Dr. Jérôme Margueron
Institut de Physique Nucléaire de Lyon, Lyon, France

1. *Purpose of the STSM*

The main scientific purpose of the STSM research project was to model effective nuclear Lagrangians from ab-initio approaches such as the Dirac-Brueckner-Hartree-Fock model for nuclear matter, based on the low-energy symmetries of QCD and the measured phase shifts from nucleon-nucleon (NN) collisions. The link between the effective Lagrangians and the more fundamental ab-initio models allows construction of microscopically constrained density-dependent relativistic Hartree-Fock (RHF) functionals that could provide more reliable and accurate description of the exotic nuclei and can be applied to describe neutron star properties in the crust as well as in the outer core.
It is therefore very interesting to study the properties of the RHF theory with respect to the covariant density functional theory (DFT), which is at present the most complex and accurate tool for description of both ground and excited states of nuclei throughout the whole periodic chart.

2. *Summary of work done during the STSM*

After initial discussions with Dr. Margueron, two lines of interest were set:
1) Investigation of the properties of exchange Fock terms in the RHF theory and their relation to the direct terms in the local Hartree approximation in order to study a possible mapping of these correlations onto a mean-field model.
2) NN phase shift analysis with respect to the connection between DBHF and DDRHF, according to the phase one in the original STSM proposal.

The first stage was dedicated to in-depth study of the mapping of Fock terms of the RHF or DDRHF theory onto the mean-field level. In this way, the local scheme can be recovered and important assets of the exchange potentials retained. This is of great interest, because underlying functionals of the simple Kohn and Sham DFT depend on the local densities only, which is one of the reasons of the simplicity and simultaneous success of the covariant mean-field models. It is therefore very actual to understand the ways how can one stay within the local scheme and in the same time retain the important aspects of exchange terms.

In order to make the approach general, I studied two different methods that lead to the inclusion of exchange effects to the Hartree level and are supposed to be complementary. In the first step, I focused on the so-called Fierz transformation, which is a type of crossing transformation that allows to express non-local Fock exchange terms in the form of local Hartree direct terms. The initial analysis revealed that within the heavy meson mass limit, which strongly simplifies the form of the meson propagator and consequently the angular exchange integrals, one can obtain Fock terms that depend only on the local densities.

This indicated the possibility of extracting local densities from the Fock terms of RHF model, provided that the saddle point approximation will be substantiated. I performed a numerical analysis in which I was able to demonstrate that the saddle point approximation, when used in angular exchange integrals of Fock terms, holds and affects the results negligibly.

In the following stage, specific forms and properties of the mapping were extracted on the level of the energy density, as well as self-energies in the scalar, time-like vector, and space-like vector channels. I studied the analytical solutions of exchange integrals in order to derive a compact and simple analytical form for the so-called exchange function $X(k_F/m_i)$, which represents an exact correction to the simple density dependence of Fock terms that was obtained using the heavy meson mass limit.

I was able to prove that when the momentum dependence of Fock self-energies is averaged over the Fermi sphere, one obtains the same exchange function as in the case of the energy density. This is an important observation, since it puts the mapping in the scalar and time-like vector channel on the unified footing. The mapping in the form of new coupling functionals has the following expression:

$$\left(\frac{G_\sigma}{m_\sigma}\right)^2 = \left(\frac{g_\sigma}{m_\sigma}\right)^2 - \left(\frac{g_\sigma}{m_\sigma}\right)^2 \frac{1}{2\gamma} X(k_F/m_\sigma) + \left(\frac{g_\omega}{m_\omega}\right)^2 \frac{2}{\gamma} X(k_F/m_\omega) + \left(\frac{g_\rho}{m_\rho}\right)^2 \frac{2(\gamma-1)}{\gamma} X(k_F/m_\rho) + \left(\frac{f_\pi}{m_\pi}\right)^2 \frac{\gamma-1}{2\gamma} X(k_F/m_\pi)$$

$$\left(\frac{G_\omega}{m_\omega}\right)^2 = \left(\frac{g_\omega}{m_\omega}\right)^2 + \left(\frac{g_\sigma}{m_\sigma}\right)^2 \frac{1}{2\gamma} X(k_F/m_\sigma) + \left(\frac{g_\omega}{m_\omega}\right)^2 \frac{1}{\gamma} X(k_F/m_\omega) + \left(\frac{g_\rho}{m_\rho}\right)^2 \frac{\gamma-1}{\gamma} X(k_F/m_\rho) + \left(\frac{f_\pi}{m_\pi}\right)^2 \frac{1-\gamma}{2\gamma} X(k_F/m_\pi)$$

Eq. 1. New effective couplings based on the proposed mapping.

To support the accuracy of the proposed mapping based on the saddle point approximation, the comparison to the full RHF calculations (standard RHF model of Bouyssy, *et al.*) was made. I studied the results on the level of energy densities and self-energies with respect to the scalar and time-like vector channels and acquired calculations were in a very good agreement [Fig 1. and 2.].

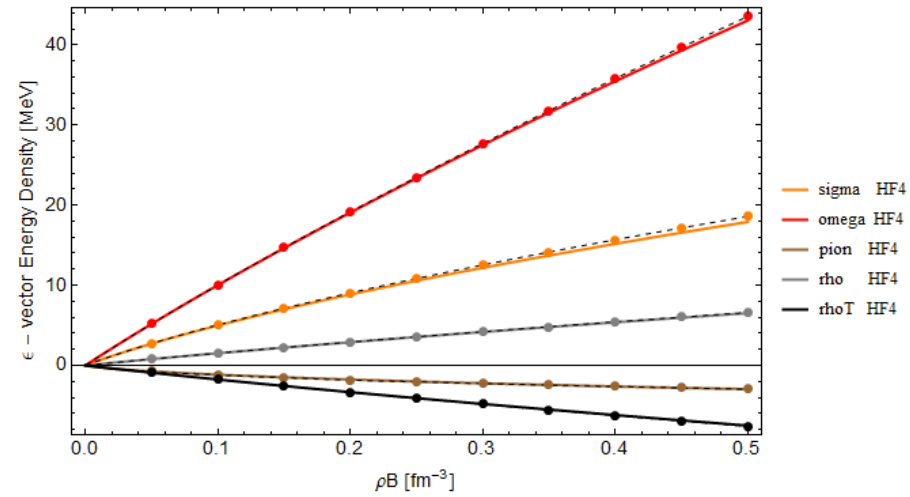

Fig. 1.
Comparison of the results of the time-like vector Fock contributions from the different meson degrees of freedom to the energy density as calculated in the RHF (full circles) and in the model based on the saddle point approximation with analytical exchange functions (lines). The dashed lines represent the full solution of the exchange integrals, while the full lines the solutions based on an approximation of the exchange function.

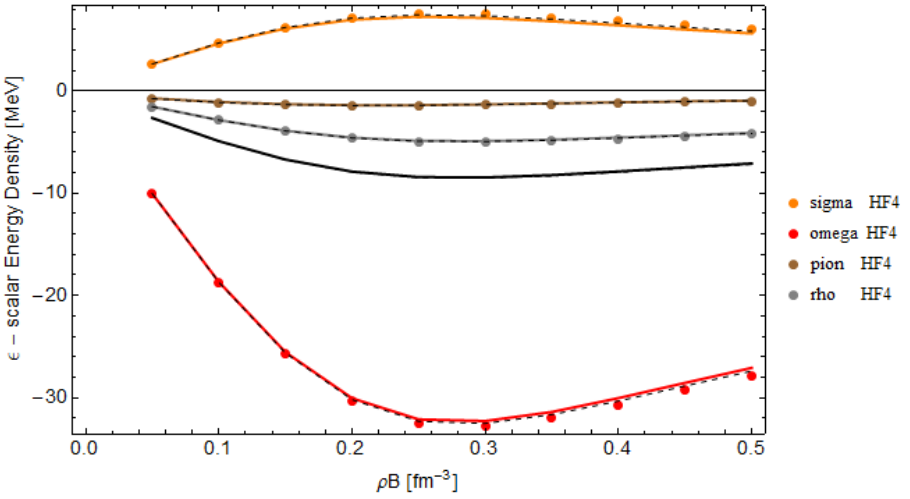

Fig. 2.
Comparison of the results of the scalar Fock contributions from the different meson degrees of freedom to the energy density as calculated in the RHF (full circles) and in the model based on the saddle point approximation with analytical exchange functions (lines). The dashed lines represent the full solution of the exchange integrals, while the full lines the solutions based on an approximation of the exchange function.

To support the accuracy of the proposed mapping based on the saddle point approximation, the comparison to the full RHF calculations was made. I studied the results on the level of energy densities and self-energies with respect to the scalar and time-like vector channels and acquired calculations were in a very good agreement [Fig 1. and 2.].

Space-like part of exchange potentials turned out to be more complicated, however, I pursued several methods to extract the mapping and reasonable agreement was obtained.

To fully explore the accuracy of the mapping, I developed a RMF model in which I incorporated the mapped exchange correlations on the level of the density-dependent couplings. Preliminary calculations were carried out and compared to the RHF model results using same degrees of freedom with neglected space-like components. The results were also compared to the standard RHF of Bouyssy, *et al. (PRC 36, No. 1, 1987)* to see the effects of the space-like correlations and the momentum dependence in self-energies. Comparison can be seen in the binding energy in Fig 3.

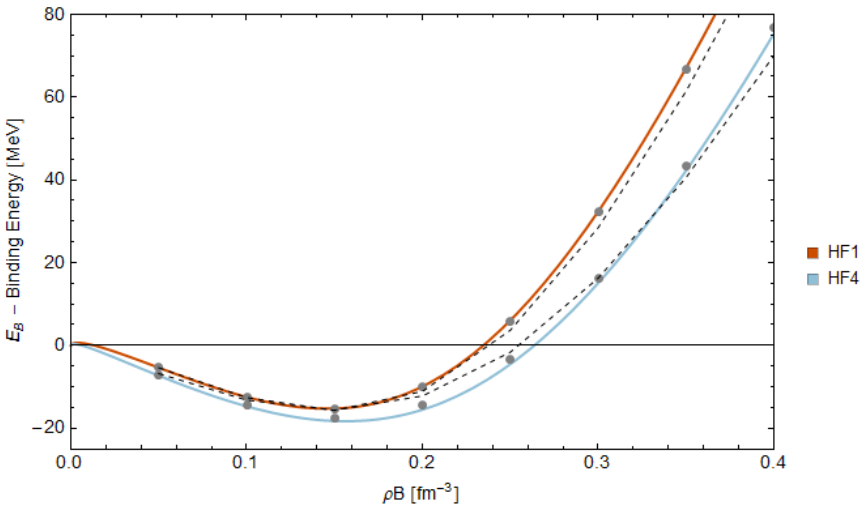

Fig. 3.
The binding energy per particle calculated in the RMF model with applied mapping of Fock terms (Full lines) as compared to the results of RHF model without the space-like vector self-energy (Full circles) and full RHF of Bouyssy, *et al.* (Dashed lines), which takes the space-like components into account. Two parameter sets from the RHF model of Bouyssy, *et al.*, were implemented - HF1 (sigma and omega mesons) and HF4 (sigma, omega, rho, pion, and rho tensor).

In conclusion, I was able to successfully derive unique mapping of the exchange correlations onto the mean-field level and test the accuracy of related approximations at the level of symmetric uniform nuclear matter. I demonstrated that it is possible to create a link between local RMF theory and non-local RHF theory through a new type of coupling functions that very accurately mimic the density dependence of Fock terms in an analytical way, and allows one to perform the calculations within the local scheme of DFT. To extend this approach to the realm of finite nuclei is of great interest and is planned for the near future.

The final phase was dedicated to the question of the link between the DDRHF and DBHF approaches. I proposed several types of effective density dependent coupling functions that could be used in the fitting procedure to obtain parametrizations of the DDRHF models. Especially, the assets and reliable behaviour of the 2-parametric class of density functions were discussed with respect to the creation of the new RHF functionals. I demonstrated an original method used to fit the DBHF data and specifics of the novel density-dependent vertex functions.

Continuation of this work is of great importance, therefore, in the following months I plan to further investigate the properties of proposed mapping with respect to the RHF and DBHF theories. New objectives are:

1) Inclusion of the density-dependent couplings and treatment of the rearrangement terms.
2) Extension of the model to the asymmetric nuclear matter.
3) Extraction of the density-dependent couplings directly from the DBHF model using the mapping.

3. *Main results of the STSM*

The main results of the STSM are as follows:

1) Mathematical derivation of the full analytical solutions of kernel integrals in RHF exchange terms.
2) A unique mapping of the exchange Fock terms of the RHF theory onto the local Hartree level has been proposed and investigated in the uniform matter.
3) A RMF model using the new type of coupling functionals based on the proposed mapping has been developed and compared to the standard RHF model with a very good agreement.
4) The 2-parametric class of density dependent functions was successfully tested to create a link between the DBHF and the DDRHF theory.

4. *Future collaboration with the host of the STSM*

The task and main objectives of the STSM project turned out to be more involved than anticipated, however, many aims have been reached, and new interesting problems investigated. As mentioned above, several new aims were defined and will be pursued in the near future.
The STSM project was very successful from the perspective of both the grantee and the host and further continuation is desired. A new STSM application will been submitted to further extend the collaboration with the Lyon group. Common scientific interests and great working relations ensure the firm cooperation in the following months.

5. *Outputs of the STSM*

Based on the obtained results and their significance, we anticipate their publication in the peer-reviewed journals in the near future. The NewCompStar (COST Action MP1304) support will be acknowledged.