COST-STSM-MP1304-29085 Report

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Purpose

The goal of the STSM was to discuss and develop simplified phenomenological lattice models for neutron stars. The motivation for such a work is based on the fact that protons in the neutron star inner crust form a lattice, and that superfluid neutrons in the crust move in such lattice. One can then aim at deriving the parameters of the lattice effective models from microscopic models and apply such models to study the behaviour of rotating stars or effects of temperature. The visit was performed together with Joao Barros, who is currently doing the PhD in Statistical physics at SISSA (Trieste), under the scientific supervision of Andrea Trombettoni.

Description of the work and main results

The work started from the discussion of different possible models for the phenomenology of neutron stars, and in particular of the inner crust. The discussions stem from a two-fold premise: from one side the experience of the Trieste components on the equilibrium and dynamical properties of fermions in lattices and periodic potentials, from the other the experience of the Coimbra components in the study of neutron stars by a variety of approaches including relativistic mean-field computations.

The main ingredients providing the rationale for the study of the STSM are the following:

- Neutrons move in the inner crust and are subjected to a single-body potential V_{1b} created by the protons;
- The potential V_{1b} is of the form of a periodic one, even though the lattice spacing and the strength intensity itself depend on the position in the crust;
- Neutrons interact with an effective scattering length which has been studied, and it is possible to encode the interactions between neutrons with a two-body potential V_{2b};
- Neutrons moving in such (idealized) lattice structure are superfluid;
- It is possible to have estimates for V_{1b} and V_{2b}, which in turn have been discussed and tested in the literature.

A first level of effective models, based on the microscopic description, is the determination of V_{1b} and V_{2b} - a further level, possibly quantitatively less accurate but easier to handle, is the determination of effective lattice models. Indeed, from many studies in condensed matter and cold atoms one knows that often effective low-energy lattice models can describe very accurately the properties of quantum particles in lattices. Once that the parameters of the effective models have been obtained, one can use them to study more complex situations.

The potential V_{1b} can be extracted from the approach developed during the years in Coimbra, see e.g. F. Grill *et al.*, *Equation of state and thickness of the inner crust of neutron stars*, Phys. Rev. C **90**, 045803 (2014) and references therein. For the determination of the lattice model and especially of the lattice parameters we use an approach similar to the one discussed in A. Trombettoni and A. Smerzi, *Discrete Solitons and Breathers with Dilute Bose-Einstein Condensates*, Phys. Rev. Lett. **86**, 2353 (2001). The starting point is a fermionic Hamiltonian for the field operator $\Psi(\vec{r}, \sigma)$ where the fermions feel a single-body potential V_{1b}

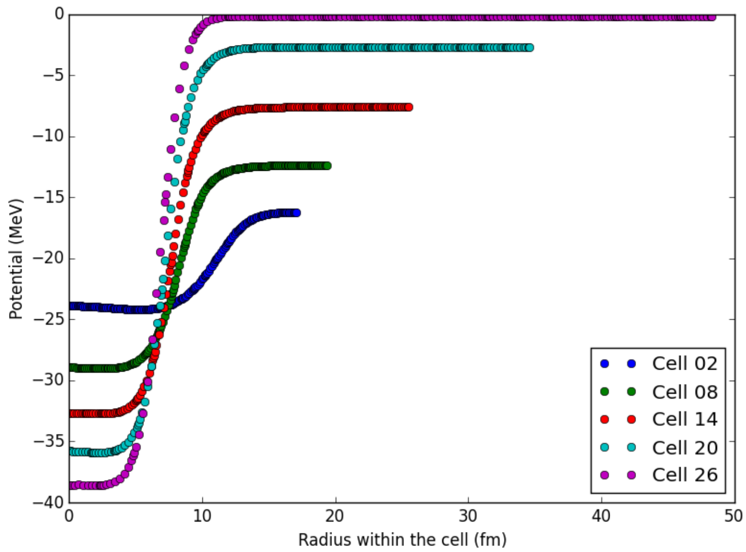

Figure 1: Potentials V_{1b} in MeV as a function of distance in fm for some of the Wigner-Seitz cells with $N = 30$ as discussed in the text. The number label of the cell is decreasing going towards the center.

and a two-body potential V_{2b}. By using a tight-binding ansatz $\Psi(\vec{r}, \sigma) = \sum_i c_{i,\sigma} \Phi_i(\vec{r})$ (where i denotes the minima of V_{1b}, in turn determined by the position of the protons, and Φ_i the Wannier wavefunction, which depends on i due to the inhomogeneity of V_{1b}) one gets an attractive Hubbard model with space-dependent parameters. The model is easier to study than the initial one, but we aimed at a greater simplification and we then used the fact that the system is superfluid: therefore its qualitative properties are described by a model in which are present the number of neutrons in the lattice cell (let denote them by n_i) and the corresponding phases φ_i. The model is then an XY model

$$H = -\sum_{\langle i,j \rangle} J_{ij} \cos(\varphi_i - \varphi_j) \tag{1}$$

where $J_{ij} = \sqrt{n_i n_j} t_{ij}$, and t_{ij} is the hopping parameter between the nearest neighbour sites i and j. The model was discussed in relation to the behaviour of vortex lines in inhomogeneous lattices in M. Iazzi *et al.*, *Vortex Lines Distribution in Inhomogeneous Lattices*, Mol. Phys. **109**, 3037 (2011).

To further proceed one has to determine, from the microscopics, the numbers n_i and the hopping parameters t_{ij}. The computation moved from the V_{1b} potential, determined in Coimbra: the results for the n_i and the t_{ij} are given in the Report by Joao Barros, and as well details on their determination. An important point has to be stressed: since one concretely wants to study the model (1), e.g. in presence of a rotation (see below), by Monte Carlo simulations, one has to fix a realistic grid. To this aim, we radially divided the inner crust in N intervals (we decided to choose $N = 30$) and compute the numbers n_i and the hopping parameters t_{ij} in the middle of such intervals. Indeed, the crust is $\sim 1km$ and the cells $\sim 10fm$, so a macroscopic number of cells are inside each interval and the potential V_{1b} may be considered not so much varying in there. The potentials V_{1b} in these N cells is reported in the Figure 1. We also notice that as soon as we focus on the model (1), the explicit knowledge of V_{2b} is not required, even though it plays a role in a more accurate determination of the t_{ij}.

Once the parameters of (1) have been determined one can think to study rotations, which is obtained via a fictitious magnetic field, as discussed in A. L. Fetter, *Rotating trapped condensates*, Rev. Mod. Phys. **81**, 647 (2009): the model (1) becomes

$$H = -\sum_{\langle i,j \rangle} J_{ij} \cos(\varphi_i - \varphi_j + A_{ij}) \tag{2}$$

where $A_{ij} = \int_i^j \vec{A} \cdot d\vec{r}$ and $\vec{A} = (m/\hbar)\vec{\omega} \times \vec{r}$, where m is the neutron mass and $\vec{\Omega} = (0, 0, \Omega)$ with Ω the angular

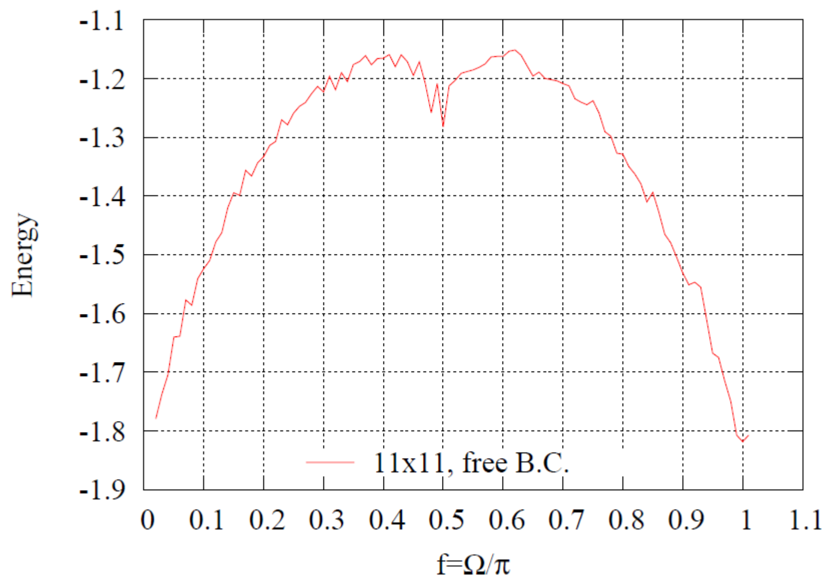

Figure 2: Energy vs. Ω for an 11×11 lattice.

velocity. The Monte Carlo code for the study of the general inhomogeneous model (2), with the parameters given as discussed, has been developed during the Coimbra visit and first results for $2D$ homogeneous lattices have been produced, as shown in Figs.2-3. Such results have been tested against Monte Carlo results available in literature.

Future

We plan to improve the estimates of the hopping parameters t_{ij} and to study in detail the $3D$ model (2) with the goal of comparing observable quantities with other phenomenological and microscopic models and with results for the neutron stars glitches.

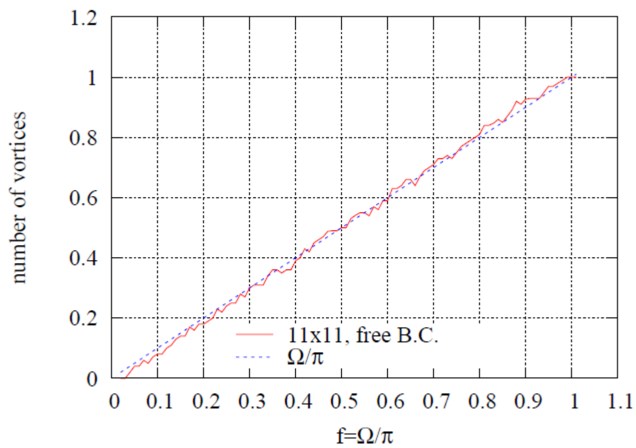

Figure 3: Number of vortices as a function of Ω for the same lattice of the previous figure.