STSM Research Report: Equation of State of Highly-Magnetised Neutron-Star Crusts

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1 Purpose of the STSM

Neutron stars are not only the most compact stars in the Universe, but they are also endowed with the strongest magnetic fields known [1]. Neutron stars thus offer the unique opportunity to explore the properties of matter under extreme conditions, which cannot be reproduced in the laboratory. Although most radio pulsars are endowed with typical surface magnetic fields of order 10^{12} G [2], a few of them have been found to have significantly higher fields of order $10^{13} - 10^{14}$ G [3]. Surface magnetic fields of order $10^{14} - 10^{15}$ G have been inferred in soft-gamma ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs) [4, 5, 6] The interior of these so called magnetars may contain even higher magnetic fields, as suggested by various observations [7, 8, 9]. Numerical simulations confirmed that magnetic fields of order $10^{15} - 10^{16}$ G can be produced during supernovae explosions due to the magnetorotational instability [10]. A huge amount of magnetic energy can be occasionally released in crustquakes leading to the observed gamma-ray bursts [11]. This scenario has been recently supported by the detection of quasiperiodic oscillations (QPOs) in the x-ray flux of giant flares. Some of these QPOs coincide reasonably well with seismic crustal modes thought to arise from the release of magnetic stresses [12, 13]. The huge luminosity variation suggests that the magnetic field is higher than 10^{15} G at the star surface thus lending support to the magnetar hypothesis [14]. Finally, internal magnetic fields of order 10^{16} G have been recently estimated from the observation of slow pulse phase modulations in AXP 4U 0142+61 [15]. According to numerical simulations, neutron stars may potentially possess internal magnetic fields as high as 10^{18} G (see, e.g. Refs. [16, 17]).

We have previously shown that the properties of the outer crust of a neutron star can be drastically modified by a sufficiently high magnetic field [18, 19].

The main goal of this STSM is to extend our investigation of highly-magnetised matter to the inner crust of a magnetar, where neutron-proton clusters coexist with free neutrons.

2 Summary of the work carried out during the STSM and preliminary results

We have implemented the effects of Landau quantization of electron motion in the semiclassical computer code developed by the Brussels-Montreal collaboration [20, 21], and based on the fourth-order extended Thomas-Fermi method with proton shell corrections added perturbatively via the Strutinsky integral theorem (neutron shell effects are very small, and can thus be neglected). This ETFSI method is a computationally high-speed approximation to the fully self-consistent Hartree-Fock+BCS equations.

In this model, nuclear clusters are supposed to be unaffected by the presence of the magnetic field, and are further assumed to be spherical. The Coulomb lattice is described following the approach of Wigner and Seitz. Nucleon density distributions in the Wigner-Seitz cell are parameterised as

$$n_q(r) = n_{B,q} + n_{\Lambda,q}\left\{1+\exp\left[\left(\frac{C_q-R_c}{r-R_c}\right)^2-1\right]\exp\left(\frac{r-C_q}{a_q}\right)\right\}^{-1} \quad (1)$$

where $q=n,p$ for neutrons or protons respectively, while $n_{B,q}$, $n_{\Lambda,q}$, C_q, a_q, and R_c are geometrical parameters of the Wigner-Seitz cell. The equation of state of nuclear clusters and free neutrons is calculated from nuclear energy density functionals based on generalised Skyrme effective interactions. For the equation of state of the magnetised electron gas, we have made use of analytical approximations as implemented in the routines developed by Potekhin and Chabrier [22]. Although our code is applicable to any temperature and composition, we shall first focus on the limiting case of cold catalysed matter. The equilibrium properties of the inner crust are determined by minimising the energy per nucleon at given average baryon number density $\bar{n}$.

We have performed various preliminary calculations to test the validity of our new code. In particular, we have carried out two series of tests using the Brussels-Montreal nuclear energy density functional BSk24 [23].

- First, we have checked that the results obtained in the limit of low magnetic fields are consistent with those obtained using the previous version of the code (some of which have been published in Ref. [24]). See Table 1.
- Second, we have checked that the neutron-drip composition and the equation of state obtained for various magnetic-field strengths are compatible with those obtained using our outer crust code [19]. As discussed in Ref. [18], we do not expect a perfect matching between the two codes since the outer and inner regions of the crust are described using different models. See Table 2.

Table 1: Comparison between inner crust equilibrium properties calculated using the older and newer versions of our ETFSI code; results for the latter code are indicated in parentheses. $B_\star$ is the magnetic field strength in units of the critical field $B_{\rm crit} = m_e^2 c^3/(e\hbar) \simeq 4.4 \times 10^{13}$ G, $\bar{n}$ is the mean baryon number density, e is the total energy per nucleon and P is the pressure.

$B_\star$	$\bar{n}$ [fm^{-3}]	Z	P [MeV fm^{-3}]	e [MeV]
0 (1)	3.03750×10^{-4}	40 (40)	5.47186×10^{-4} (5.46553×10^{-4})	-1.52388×10^{0} (-1.52388×10^{0})
0 (1)	6.15788×10^{-4}	40 (40)	8.09376×10^{-4} (8.08524×10^{-4})	-5.05439×10^{-1} (-5.05438×10^{-1})
0 (1)	3.20307×10^{-3}	40 (40)	4.42962×10^{-3} (4.42734×10^{-3})	1.45530×10^{0} (1.45530×10^{0})
0 (1)	1.48098×10^{-2}	40 (40)	3.25210×10^{-2} (3.25113×10^{-2})	4.11657×10^{0} (4.11657×10^{0})
0 (1)	4.27486×10^{-2}	40 (40)	1.12867×10^{-1} (1.12837×10^{-1})	6.64811×10^{0} (6.64811×10^{0})
0 (1)	6.60442×10^{-2}	40 (40)	2.01108×10^{-1} (2.01067×10^{-1})	7.83743×10^{0} (7.83743×10^{0})

Table 2: Comparison between the outer-crust and inner-crust codes at the neutron-drip point; results for the latter code are indicated in parentheses. $\bar{n}_{drip}$ and $P_{\rm drip}$ are mean baryon number density and the pressure at the neutron drip-point respectively.

$B_\star$	$\bar{n}_{drip}$ [fm^{-3}]	Z	N	$P_{\rm drip}$ [MeV fm^{-3}]
1	2.564538×10^{-4}	38 (40)	86 (94)	5.13570×10^{-4} (4.95927×10^{-4})
5	2.564428×10^{-4}	38 (40)	86 (94)	5.13540×10^{-4} (4.95911×10^{-4})
10	2.564082×10^{-4}	38 (40)	86 (94)	5.13448×10^{-4} (4.95858×10^{-4})
100	2.567754×10^{-4}	38 (40)	86 (94)	5.14429×10^{-4} (4.96416×10^{-4})
500	2.604989×10^{-4}	38 (40)	86 (94)	5.63685×10^{-4} (5.39812×10^{-4})
1000	2.913752×10^{-4}	38 (41)	86 (96)	6.87034×10^{-4} (6.64014×10^{-4})
1500	2.197913×10^{-4}	38 (40)	86 (92)	8.76236×10^{-4} (8.51014×10^{-4})
2000	2.945235×10^{-4}	38 (41)	86 (95)	1.18058×10^{-3} (1,14499×10^{-3})

3 Future collaboration with the host of the STSM

After all the tests are completed, we plan to perform systematic calculations of the equation of state of the outer and inner regions of magnetar crusts using the recent series of accurately calibrated Brussels-Montreal nuclear energy density functionals. This will allow us to assess the uncertainties associated with the lack of knowledge of high-density matter. Calculations will be carried out for the range of magnetic-field strengths required for modelling astrophysical phenomena related to magnetars. The results of this project will be presented at international scientific conferences, and scientific papers will be submitted to peer-reviewed international journals.

References

[1] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars 1: Equation of state and structure*, Springer (2007).

[2] J. H. Seiradakis and R. Wielebinski, Astron.Astrophys.Rev.12,239(2004).

[3] C. Y. Ng and V. M. Kaspi, AIP Conf. Proc.**1379**, 60 (2011).

[4] S. A. Olausen & V. M. Kaspi, Astrophys. J. Suppl. Ser. **212**, 6 (2014).

[5] A. Tiengo et al., Nature **500**, 312 (2013).

[6] Hongjun An et al., Astrophys. J. **790**, 60 (2014).

[7] L. Stella, S. DallOsso, and G. L. Israel, Astrophys. J.**634**, L165 (2005).

[8] A. D. Kaminker, D. G. Yakovlev, A. Y. Potekhin, N. Shibazaki, P. S. Shternin, O. Y. Gnedin, Astrophys. Sp. Sci.**308**, 423 (2007).

[9] N. Rea, P. Esposito, R. Turolla, G. L. Israel, S. Zane, L. Stella, S. Mereghetti, A. Tiengo, D. Gotz, E. Gogus and C. Kouveliotou, Science **330**, 944 (2010).

[10] N. V. Ardeljan, G. S. Bisnovatyi-Kogan, S. G. Moiseenko, Mon. Not. Roy. Astr. Soc.**359**, 333 (2005).

[11] C. Thompson and R. C. Duncan, Mon. Not. Roy. Astr. Soc.**275**, 255 (1995).

[12] A. L. Watts and T. E. Strohmayer, Adv. Sp. Res. **40**, 1446 (2007).

[13] L. Samuelsson and N. Andersson, Mon. Not. Roy. Astr. Soc. **374**, 256 (2007).

[14] M. Vietri, L. Stella, G. L. Israel, Astrophys. J. **661**, 1089 (2007).

[15] K. Makishima, T. Enoto, J. S. Hiraga, T. Nakano, K. Nakazawa, S. Sakurai, M. Sasano, H. Murakami, Phys. Rev. Lett. **112**, 171102 (2014).

[16] A. G. Pili, N. Bucciantini, L. Del Zanna, Mon. Not. R. Astron. Soc. **439**, 3541 (2014).

[17] D. Chatterjee, T. Elghozi, J. Novak, M. Oertel, Mon. Rot. R. Astron. Soc. **447**, 3785 (2015).

[18] N. Chamel, R. L. Pavlov, L. M. Mihailov, Ch. J. Velchev, Zh. K. Stoyanov, Y. D. Mutafchieva, M. D. Ivanovich, J. M. Pearson, and S. Goriely, Phys. Rev. C **86**, 055804 (2012).

[19] N. Chamel, Zh. K. Stoyanov, L. M. Mihailov, Y. D. Mutafchieva, R. L. Pavlov, and Ch. J. Velchev, Phys. Rev. C **91**, 065801 (2015).

[20] J. M. Pearson, N. Chamel, S. Goriely, and C. Ducoin, Phys. Rev. C **85**, 065803 (2012).

[21] J. M. Pearson, N. Chamel, A. Pastore, and S. Goriely, Phys. Rev. C **91**, 018801 (2015).

[22] A. Y. Potekhin, G. Chabrier, Astron. Astrophys. **550**, A43 (2013); `http://www.ioffe.ru/astro/EIP/`

[23] S. Goriely, N. Chamel, and J. M. Pearson, Phys. Rev. C **88**, 024308 (2013).

[24] J. M. Pearson, N. Chamel, A. F. Fantina, and S. Goriely, Eur. Phys. J. A **50**, 43 (2014).