Dr. Kristian Petrík
Institute of Physics, Slovak Academy of Sciences
Department of Nuclear Physics
Dúbravská cesta 9
845 11 Bratislava, Slovakia

STSM 2016 RESEARCH PROJECT REPORT

Investigation and development of the novel mapping of the density-dependent relativistic Hartree-Fock theory onto the mean-field model.

ECOST-STSM-MP1304-020516-077190
02/05/2016 – 10/06/2016
Host: Dr. Jérôme Margueron
Institut de Physique Nucléaire de Lyon, Lyon, France

1. *Purpose of the STSM*

The main scientific purpose of this project is to study the *localized Fock approximation (LFA)*, which allows one to express the Fock terms of a density-dependent relativistic Hartree-Fock (DDRHF) theory in a form that can be projected onto a mean-field Hartree theory. In this way a new type of coupling functionals can be introduced, which take into account the non-local exchange correlations, but respect the structure of the mean-field theory. The focus will be put on the properties of nuclear matter within the density-dependent approaches with PKO1-3 and PKA1 parametrizations. We will test the quality of the underlying saddle-point approximation with respect to the energy density, self-energies, rearrangement and other important quantities, which are crucial for the successful application of the mapping.

This project represents a natural continuation of the STSM project COST-STSM-ECOST-STSM-MP1304 261015-063615 that finished in 2015.

2. *Summary of work done during the STSM*

In the first stage of the current STSM project we focused on the implementation of our *localized Fock approximation* in the density dependent class of relativistic Hartree-Fock models that use the well-known PKA1 and PKO1-3 parametrizations. The LFA has been previously used with success in the RHF models with coupling constants, however, the more modern approaches are of great interest for the scientific community.

Studying the exchange part of the nuclear interaction, which is represented by exchange Fock terms in the RHF energy density expressions, one can see that the scalar, time-like and space-like vector channels contain the following integrals:

$$I_F = \int_0^{k_F} kdk \int_0^{k_F} qdq \ F(k,q),$$
$$J_F = \int_0^{k_F} kdk\hat{M}(k) \int_0^{k_F} qdq\hat{M}(q) \ F(k,q),$$
$$K_F = \int_0^{k_F} kdk\hat{P}(k) \int_0^{k_F} qdq\hat{P}(q) \ F(k,q),$$
$$L_F = \int_0^{k_F} kdk\hat{P}(k) \int_0^{k_F} qdq\hat{M}(q) \ F(k,q),$$

These integrals, here expressed in a general way, enter the self-consistent calculations and produce the density dependence of non-local Fock correlations.

Depending on the used parametrization some of the integrals are not taken into account, nonetheless, our description will be as general as possible. Functions F(k,q) are momentum dependent and represent the angular exchange functions of the RHF approach that result from the meson propagators.

In the rather limiting *heavy meson mass limit* the propagators are simplified in such way that the complicated exchange integrals can be transformed to a simple functions of local densities (in the case of the energy density, the densities will be squared). This hints on the possibility to turn the exchange terms in such form that resembles the mathematical form of the Hartree direct terms. Our aim was therefore to simplify the exchange Fock terms in order to extract the relevant densities from the integrals, however, without sacrificing the accuracy of our model. Taking everything into account, we proposed the transformation rule of LFA:

$$J_F \rho_B^2 \approx I_F \rho_S^2,$$
$$K_F \rho_B^2 \approx I_F \rho_V^2,$$
$$L_F \rho_B^2 \approx I_F \rho_V \rho_S.$$

The coupled integrals J_F, K_F and L_F are thus expressed through the analytically solvable integral I_F multiplied by the square of the relevant density for given channel – baryon density ρB, scalar density ρS and the new space-like vector density ρV.

Similar rule was derived for the self-energies, where we considered the momentum independent as well as momentum dependent case. Although the effects of the momentum dependency of self-energies are not very crucial in combined quantities, like binding energy, or total scalar and vector self-energies, the differences between approaches with and without momentum dependence play an important role when the finite nuclei or separate channels of each meson are considered.

On the theoretical level, the implementation of the LFA to DDRHF models was straightforward and we have successfully finished many calculations for the symmetric matter case with respect to all parametrizations in consideration.

One of our objectives was related to creating a mapping between RHF model and mean-field Hartree model. This can be done by expressing the new space-like vector density ρV in terms of the typical mean-field densities ρB and ρS according to the this approximation

$$\rho_V^2 \approx \rho_B^2 - \rho_S^2.$$

In this way, the contributions coming from the space-like vector terms can be mapped to the scalar and time-like vector channels and hence the mean-field like model can be recovered.
In the following, we would like to discuss some of the most important findings.

In Fig 1. And Fig 2., the binding energy of symmetric nuclear matter is shown for some of the used parametrizations. One can see that the reproduction of the underlying DDRHF model (points) by our RHF-LFA model is excellent in the whole density range.

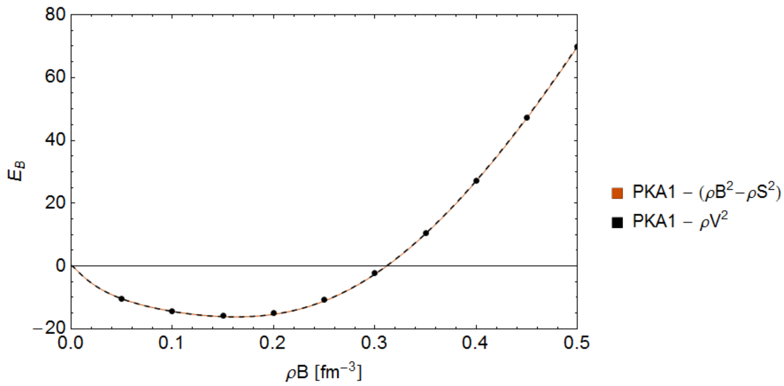

Fig 1.

Comparison of the binding energy results as calculated in the DDRHF with PKA1 (full circles) and in our RHF-LFA approach based on the saddle-point approximation (dashed line).

The red line represents the solution of the mapped model RHmap that is based on the transformation of the space-like vector density ρV.

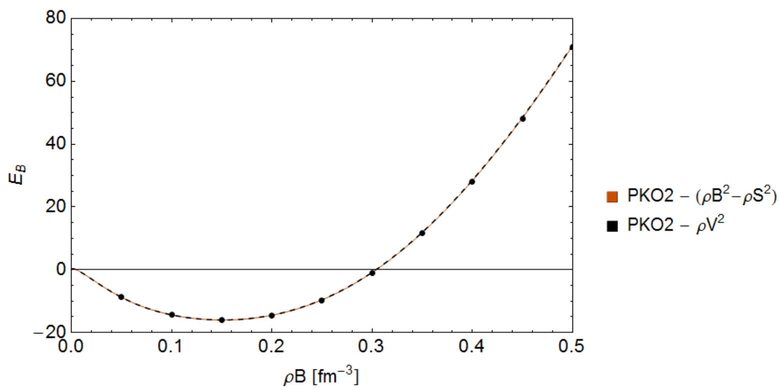

Fig 2.

Comparison of the binding energy results as calculated in the DDRHF with PKO2 (full circles) and in our RHF-LFA approach based on the saddle-point approximation (dashed line).

The red line represents the solution of the mapped model RHmap that is based on the transformation of the space-like vector density ρV.

Similarly good results have been obtained for all parametrizations in the PKO series.

If we look on the reproduction of the DDRHF data in separate channels and meson types in Fig 3., we can see the very good agreement as well. The similar quality of the reproduction can be observed for all mesons in consideration, with only minor differences that sometimes occur in higher densities and indicate the limits of used approximations. However, the results for symmetric matter are excellent in general.

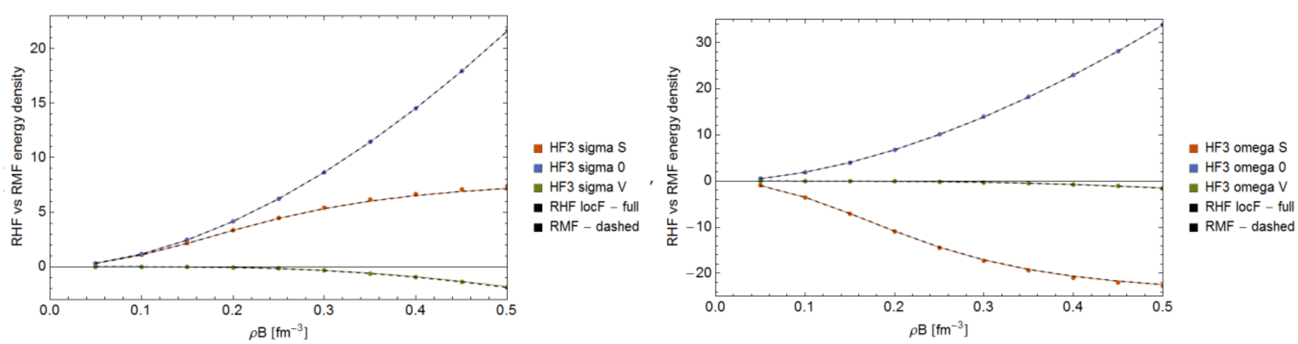

Fig 3.

Sigma and omega meson contribution to the energy density separated into scalar and vector channels. Full circles represent the DDRHF data, coloured full lines the RHF-LFA model and dashed black lines the mapped RHmap model.

Another important feature of all density dependent approaches is the rearrangement contribution to the time-like self-energy that results from the fact that the couplings are functions of the baryon density. Moreover, our LFA transforms the exchange Fock terms in a way that introduces another dependence on the baryon density, which has to be taken into account when deriving the full rearrangement contributions.

We have studied the rearrangement in the case of PKA1 and PKO1-3 parameters and acquired results that confirm the thermodynamic consistency of all approaches based on the proposed *localized Fock approximation*. In Fig 4., the results for PKO1 model are shown. If the theory is thermodynamically consistent (satisfies the Hugenholtz-Van Hove theorem in nuclear matter), the chemical potential per particle should cross the binding energy at saturation density. We have proved that the rearrangement contributions calculated within our models for all parametrizations comply this rule.

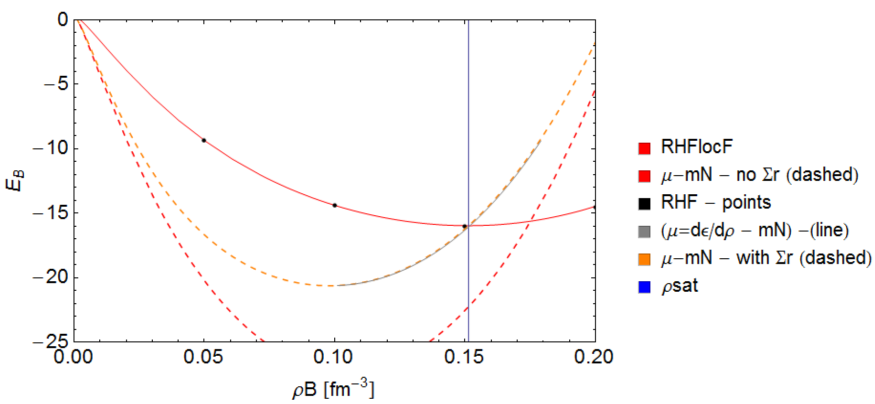

Fig 4.
Results with PKO1 parametrization are shown to indicate the preservation of the thermodynamic consistency of RHF-LFA approach. The red full line represents the binding energy of our model and red dashed line the chemical potential without the rearrangement contribution. The orange dashed line that crosses the binding energy at saturation indicates the chemical potential corrected by the rearrangement self-energy.

In conclusion, all objectives of the STSM have been successfully fulfilled. Additionally, some preliminary analyses of the extended RHF-LFA model for finite nuclei were carried out and show promising results. Also, the extension to asymmetric nuclear matter is currently under investigation and first results show the same level of agreement with RHF approach as in the case of symmetric matter. We can conclude that this mission has been a success beyond expectation.

3. *Main results of the STSM*

The main results of the STSM are as follows:

1) We have studied the effects of the newly invented *localized Fock approximation (LFA)* based on the saddle-point approximation within the DDRHF models with PKA1 and PKO1-3 parametrizations [W.-H. Long, N. van Giai, and J. Meng, Phys. Lett. B, 150 (2006), W.-H. Long, *et al.*, Phys. Rev. C 76, 034314 (2007)]. Results show that our RHF model in the LFA is extremely successful in reproducing the original DDRHF symmetric matter properties, like the binding energies, energy density in all channels, as well as the total self-energies. This justifies the proposed localization procedure of the RHF approach at the level of nuclear matter.

2) We have investigated the rearrangement contribution to the time-like vector self-energy that stems from the density dependence of studied approaches and is necessary to retain the thermodynamic consistency of the framework. We have recovered the correct rearrangement terms that come from both the density dependence of coupling functions as well as the inherent density dependence of Fock correlations. By comparing the chemical potential and binding energies for all considered parametrizations we have proved that our approach is thermodynamically consistent.

4. *Future collaboration with the host of the STSM*

The task and main objectives of the current STSM project have been fully accomplished, furthermore, many other, previously unanticipated and new interesting problems have been investigated (extensions of our approach to the asymmetric nuclear matter, finite nuclei studies within the local density approximation and WKB approximation, connections between the DBHF theory and RHF effective Lagrangian, to name a few). The STSM project was very successful from the perspective of both the grantee and the host and further continuation is desired and will be pursued. Common scientific interests and great working relations ensure the firm cooperation in the future and guarantee high quality publications.

5. *Outputs of the STSM*

Based on the obtained results and their significance, we anticipate their publication in the peer-reviewed journals in the near future. We are currently finishing the first publication from a planned series of articles related to the proposed model by our group. This publication is focused on the introduction of *localized Fock approximation* in RHF model with coupling constants. Second publication will be based on the results implemented in the finite nuclei calculations that employ the *local density approximation* and *WKB approximation*. The following publications will cover the extensions to the density-dependent RHF models and fully asymmetric nuclear matter framework, which allows one to explore the effects of Fock correlations on the properties of neutron and hyperon stars. This is an important step, since the majority of current compact star models rely on the Hartree approximation only. Effects of exchange Fock contributions could provide new insights on the character of nuclear interaction within the macroscopic astrophysical systems.